On Pairwise Compatibility Graphs having Dilworth Number Two.[1]

T. Calamoneri, R. Petreschi

Department of Computer Science, "Sapienza" University of Rome, Italy
Email: calamo@di.uniroma1.it, petreschi@di.uniroma1.it

Abstract

A graph $G = (V, E)$ is called a *pairwise compatibility graph (PCG)* if there exists a tree T, a positive edge-weight function w on T, and two non-negative real numbers d_{min} and d_{max}, $d_{min} \leq d_{max}$, such that V coincides with the set of leaves of T, and there is an edge $(u, v) \in E$ if and only if $d_{min} \leq d_{T,w}(u, v) \leq d_{max}$ where $d_{T,w}(u, v)$ is the sum of the weights of the edges on the unique path from u to v in T. When the constraints on the distance between the pairs of leaves concern only d_{max} or only d_{min} the two subclasses LPGs (Leaf Power Graphs) and mLPGs (minimum Leaf Power Graphs) are defined.

The *Dilworth number* of a graph is the size of the largest subset of its nodes in which the close neighborhood of no node contains the neighborhood of another.

It is known that $LPG \cap mLPG$ is not empty and that threshold graphs, i.e. Dilworth one graphs, are contained in it. In this paper we prove that Dilworth two graphs belong to the set $LPG \cap mLPG$, too. Our proof is constructive since we show how to compute all the parameters T, w, d_{max} and d_{min} exploiting the usual representation of Dilworth two graphs in terms of node weight function and thresholds. For graphs with Dilworth number two that are also split graphs, i.e. split permutation graphs, we provide another way to compute T, w, d_{min} and d_{max} when these graphs are given in terms of their intersection model.

Keywords: pairwise compatibility graphs, Dilworth number, leaf power graphs, minimum leaf power graphs, threshold signed graphs, split permutation graphs,

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interval graphs.

1. Introduction

A graph $G = (V, E)$ is a *pairwise compatibility graph* (PCG) if there exists a tree T, a positive edge-weight function w on T and two non-negative real numbers d_{min} and d_{max}, $d_{min} \leq d_{max}$, such that V coincides with the set of leaves of T, and there is an edge $(u, v) \in E$ if and only if $d_{min} \leq d_{T,w}(u, v) \leq d_{max}$ where $d_{T,w}(u, v)$ is the sum of the weights of the edges on the unique path from u to v in T. In such a case, we say that G is a PCG of T for d_{min} and d_{max}; in symbols, $G = PCG(T, w, d_{min}, d_{max})$. In Fig.1 an example of PCG is depicted.

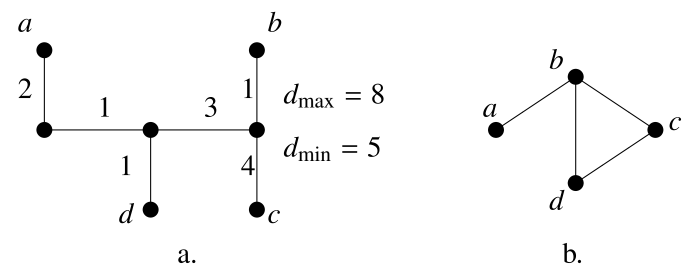

Figure 1: a. A pairwise compatibility tree; b. the corresponding pairwise compatibility graph.

The *pairwise compatibility graph recognition problem* consists in determining whether a given graph is PCG or not while the *pairwise compatibility tree construction problem* consists in finding out a tree T, an edge-weight function w and two suitable values, d_{min} and d_{max}, such that the given graph G is $PCG(T, w, d_{min}, d_{max})$.

Since 2003, when the PCG class was introduced by Kearney et al. [16] dealing with a sampling problem in a phylogenetic tree, this class of graphs has received great interest from researchers belonging to different fields, from computational biology to computational complexity and graph theory. Indeed, the main application of the pairwise compatibility graph recognition problem remains in phylogenetics, in relation to the reconstruction of a tree expressing the evolutionary relations among organisms based on quantitative biological data [14]. Nevertheless, researchers interested in computational complexity theory are fascinated by these graphs because the clique problem is known to be polynomially solvable for PCGs once the pairwise compatibility tree is provided [16] (while it is well known that, for a general graph, finding whether there is a clique of a given size is NP-complete [13]). Moreover, the class of PCGs appears interesting by itself from a graph theoretic point of view; nowadays, there are a few results proving that some special classes of graphs are PCGs [1, 15, 17, 19, 20] but only two results for general graphs, one affirming that all graphs with a number of nodes not greater than 7 are PCGs [6, 18] and the other one proving that not all graphs are PCGs, by means of two graphs that cannot be a PCG, one with 15 nodes [20] and the other one with 8 nodes [11].

One of the most recent results concerned with this class of graphs [8] explores the relation between the PCG class and two particular subclasses resulting from the cases where the constraints on the distance between the pairs of leaves deal only with d_{max} (LPG) or only with d_{min} (mLPG). More precisely, a graph $G = (V, E)$ is called a *leaf power graph*, LPG (respectively *minimum leaf power graph*, mLPG) if there exists a tree T, a positive edge-weight function w on T, and a non-negative real number d_{max} (respectively d_{min}) such that V coincides with the set of leaves of T, and there is an edge $(u, v) \in E$ if and only if $d_{T,w}(u, v) \leq d_{max}$ (respectively $d_{T,w}(u, v) \geq d_{min}$), where $d_{T,w}(u, v)$ is the sum of the weights of the edges on the unique path from u to v in T. In [8] the authors show that the union of these two proper subclasses does not coincide with the whole PCG class and that neither of the classes LPG and mLPG is contained in the other. Moreover, the class of threshold graphs is in both LPG and mLPG.

Threshold graphs correspond to graphs with Dilworth number one [10, 12], where the *Dilworth number* of a graph is the size of the largest subset of its nodes in which the close neighborhood of no node contains the neighborhood of another. In this paper we consider the wider class of graphs with Dilworth number at most two proving that they are a subclass of $LPG \cap mLPG$, so progressing toward the characterization of this intersection.

The rest of this paper is organized as follows: Section 2 lists some notions that are useful for the forthcoming work. The first result presented in this work is detailed in Section 3, where it is shown that graphs with Dilworth number two are in $LPG \cap mLPG$ providing a method to find T, w, d_{max} and d_{min}. In Section 4 our attention is focused on split permutation graphs, i.e. the subclass of graphs with Dilworth number two that are also split graphs. The intersection model of this subclass is exploited to give another way for finding out T, w, d_{min} and d_{max}; to this aim we introduce two novel transformations that, starting from the permutation diagram of a split permutation graph, produce the interval intersection model of the same graph and the permutation diagram of its complement.

The last section proposes some conclusions and open problems arisen from this work.

2. Preliminary definitions and properties

In this section we introduce some terminology and recall some definitions that will be used in the rest of the paper. The reader is referred to [2] for undefined terms and notation. We consider only simple and connected graphs $G = (V, E)$ with node set V and edge set E.

A graph $G = (K, I, E)$ is said to be *split* if there is a node partition $V = K \cup I$ such that the subgraph induced by K is a *clique*, while the subgraph induced by I is a *stable set*.

For each node v in a graph $G = (V, E)$, we call its *open neighborhood* the set $N(v) = \{u | (u, v) \in E\}$ and its *closed neighborhood* the set $N[v] = N(v) \cup \{v\}$. Two nodes x and y are said to be *comparable* if either $N(y) \subseteq N[x]$ or $N(x) \subseteq N[y]$. This relation is reflexive and transitive, but it is not antisymmetric as a graph may contain distinct nodes with the same neighborhood. A *chain* is a set of pairwise comparable nodes and the Dilworth number of a graph is the largest number of pairwise incomparable nodes of the graph or, in other words, the minimum size of a partition of its nodes into chains.

A graph $G = (V, E)$ is a *threshold graph* if there is a positive real number S (the threshold) and for every node v there is a real weight $a(v)$ such that (v, w) is an edge if and only if $a(v) + a(w) \geq S$. Threshold graphs are completely determined by their set of nodes V, threshold S and node-weight function a, so we indicate them as $G = (V, a, S)$. Threshold graphs coincide with Dilworth one graphs [10], while graphs with Dilworth number at most two coincide with a generalization of threshold graphs, called threshold signed graphs [4].

A graph $G = (V, E)$ is a *threshold signed graph* [4] if there are positive real numbers S, T (the thresholds) and for every node v there is a real weight $a(v) < min(S, T)$ such that (v, w) is an edge if and only if either $|a(v) + a(w)| \geq S$ or $|a(v) - a(w)| \geq T$. In Figure 2.a a threshold signed graph is depicted. Notice that if $S = T$ then the threshold signed graph is simply a threshold graph. For any node v, if $a(v) = 0$ then v is an isolated node, so if (u, v) is an edge in a connected graph G, then $a(u) \cdot a(v) \neq 0$. Consider an edge (u, v) of a threshold signed graph. It is not difficult to see that only one of the two conditions concerning the thresholds can be satisfied; so, if $a(u) \cdot a(v) > 0$, meaning that $a(u)$ and $a(v)$ have the same signs, then it must be that $|a(u) + a(v)| \geq S$ and the edge (u, v) is called *S-edge*; if, on the contrary, $a(u) \cdot a(v) < 0$, that is $a(u)$ and $a(v)$ have different sign, then it must be that $|a(u) - a(v)| \geq T$ and the edge is called *T-edge*. We call E_S and E_T the sets of S- and T-edges. Of course, $E = E_S \cup E_T$. We can consider the partition of the nodes of G into two sets X and Y such that $X = \{x \in V \text{ s.t. } a(x) < 0\}$ and

$Y = \{y \in V \text{ s.t. } a(y) > 0\}$. As consequence, (u, v) is an *S-edge* (*T-edge*) if u and v are in the same (different) set X or Y.

Finally, we remark that the graph $G_S = (V, E_S)$ is exactly the union of two node-disjoint threshold graphs, one corresponding to X and one to Y and, consequently, X and Y are two node-disjoint chains, while the graph $G_T = (V, E_T)$ is a bipartite graph [2]. From now on we will refer to a threshold signed graph as $G = (V, a, S, T)$ always assuming $V = X \cup Y$ and $S > T$, according to the construction presented in [4].

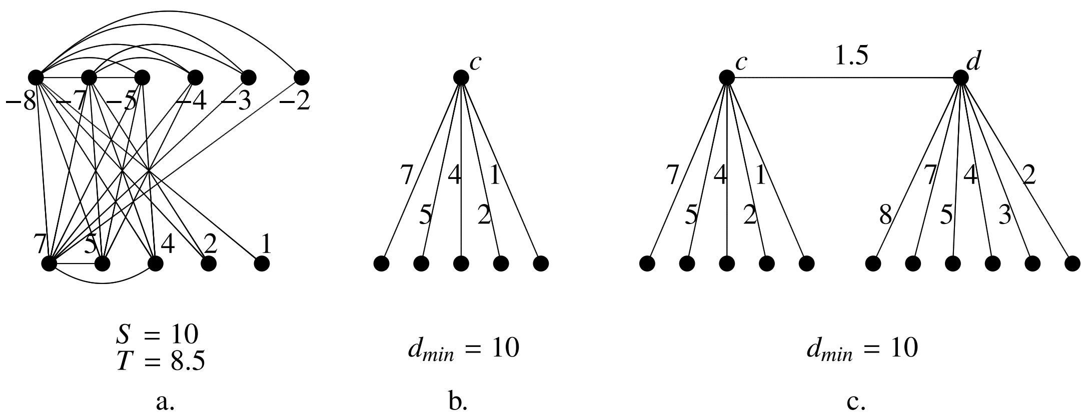

Figure 2: a. A threshold signed graph G; b. the star S_5 that witnesses that the graph induced by the five nodes with positive weight (set Y in the text) is in mLPG; c. the caterpillar C that witnesses that G is in mLPG.

Given a graph $G = (V, E)$, its *complement* G^C has the same node set V of G and two nodes are adjacent if and only if they are not adjacent in G.

The class of threshold signed graphs is closed under complement [4].

Interval graphs are the intersection graphs of closed intervals on the real line. The complement of an interval graph is called *co-interval graph* [2].

A *caterpillar* is a tree in which all the nodes are within distance one of a central path which is called the *spine*.

3. Graphs with Dilworth Number Two are in *LPG* ∩ *mLPG*

In this section we show that any threshold signed graph is both a mLPG and a LPG exploiting a construction of the edge-weighted witness tree for threshold graphs. The known construction for threshold graphs [8] uses the definition – among the numerous equivalent ones of threshold graphs – based on the degree sequence; here we recall that construction by proposing a slight modification, in order to use the definition founded on the threshold S and the node-weight function a. Informally speaking, in this way we transform a node-weight construction (on G) into an edge-weight one (on T).

Construction for Threshold Graphs. The pairwise compatibility tree of a threshold graph $G = (V, a, S)$ is a star S_n with $n+1$ nodes (a central node c and leaves $v_1, \ldots, v_n$); the weight of each edge (c, v_i), $w(c, v_i)$, is equal to $a(v_i)$ (in Figure 2.b it is shown the star S_5 corresponding to the threshold graph induced by the nodes with positive weights of the graph in Figure 2.a); it can be proven that $G = mLPG(S_n, w, S)$. It is to notice that it is also possible to construct an analogous edge-weighted star showing that the threshold graph is in LPG. □

Now we show how to build a tree C that is a caterpillar, an edge-weight function w and two values d_{min} and d_{max} for showing that a threshold signed graph is both an mLPG and a LPG.

Lemma 1. *Let $G = (V, a, S, T)$ be a threshold signed graph, with $V = X \cup Y$ and $S > T$, for S, T positive real numbers. A caterpillar C with edge-weight function w and a value d_{min} such that $G = mLPG(C, w, d_{min})$ can be found in polynomial time.*

Proof. Let us focus on the two threshold graphs induced by Y and X. For them we construct two stars, $S_{|Y|}$ (whose central node is c) and $S_{|X|}$ (whose central node is d) using the construction given above with the only exception that when we handle the threshold graph induced by X, for each node $v \in X$ we assign $-a(v)$ to the edge incident to the leaf corresponding to v. In order to construct the caterpillar C, connect the central nodes c and d of stars $S_{|X|}$ and $S_{|Y|}$ by means of an edge with weight $w(c, d) = S - T$. This value is always positive, as $S > T$ (see Figure 2.c).

Now we have to prove that the mLPG generated by C with $d_{min} = S$ is exactly the given threshold signed graph G. Consider any edge (u, v) of G. If u and v are both in X or both in Y, then $|a(u) + a(v)| \geq S$ and hence we have that the length of the edge-weighted path on C is $d_C(u, v) = w(x, u) + w(x, v) = |a(u) + a(v)| \geq S$, (with either $x = c$ or $x = d$) so proving that (u, v) is also an edge of the mLPG. If $u \in Y$ and $v \in X$ then it holds $|a(u) - a(v)| \geq T$. Recalling that the nodes in X have negative weights while the nodes in Y have positive weights, the length of the corresponding path in C is $d_C(u, v) = w(u, c) + w(c, d) + w(d, v) = a(u) + S - T - a(v) = |(a(u) - a(v))| + S - T \geq T + S - T = S$ hence (u, v) is also an edge of the mLPG. Let us now consider in G two not connected nodes u and v, then the following two inequalities hold: $|a(u) + a(v)| < S$ and $|a(u) - a(v)| < T$. If u and v belong to the same set, their distance on C is $d_C(u, v) = w(x, u) + w(x, v) = |a(u) + a(v)| < S$, (with either $x = c$ or $x = d$), and hence (u, v) is not an edge of the mLPG. If, on the contrary, $u \in Y$ and $v \in X$, then $d_C(u, v) = w(u, c) + S - T + w(d, v) =$

$a(u) + S - T - a(v) = |(a(u) - a(v))| + S - T < T + S - T = S$ and hence, also in this case, (u, v) is not an edge of the mLPG.

So the proof is concluded since we know that $mLPG(C, w, S)$ and G have the same number of nodes, and we have proved that $mLPG(C, w, S)$ contains all edges of G and only those. □

Proposition 1. *[7] The complement of every graph in LPG is in mLPG and conversely, the complement of every graph in mLPG is in LPG. In particular, if* $G = LPG(T, w, d_{max})$ *then* $G^C = mLPG(T, w, \min_{\{u,v\}\notin E(G)} d_T(u, v))$*; if* $G = mLPG(T, w, d_{min})$ *then* $G^C = LPG(T, w, \max_{\{u,v\}\notin E(G)} d_{T,w}(u, v))$.

Lemma 2. *Let G be a threshold signed graph* $G = (X \cup Y, a, S, T)$ *and* $S > T$*. There exists a caterpillar C with edge-weight function w and a value* d_{max} *such that* $G = LPG(C, w, d_{max})$.

Proof. Consider the graph complement of G, G^C. As the class of threshold signed graphs is self-complemented, G^C is also a threshold signed graph, and it is possible to determine its edge-weight function w and the two thresholds S and T, with $S > T$. Apply now the construction of Lemma 1 to find an edge-weighted caterpillar C and a value d_{min} so that $G^C = mLPG(C, w, d_{min})$. From Proposition 1, it follows that the graph complement of G^C is $G = LPG(C, w, d_{max})$, where $d_{max} = \max_{\{u,v\}\notin E(G)} d_{T,w}(u, v)$. □

Lemmas 1 and 2 easily imply the following result:

Theorem 1. *The class of graphs with Dilworth number at most two is in LPG ∩ mLPG.*

4. Split Permutation Graphs

In this section we focus on *split permutation graphs*, i.e. the subclass of graphs with Dilworth number two that are also split graphs. For this class, of course, Theorem 1 holds, but we present another way to find out T, w, d_{min} and d_{max} that can be used when G is given not by means of sets X, Y and by thresholds S, T, but by means of its intersection model. To this aim, we first detail the *intersection model* (also called *permutation diagram*) of split permutation graphs.

A *permutation* π on the set $\{1, \ldots, n\}$ is a bijection from the set to itself. A commonly used way of representing a permutation is the so called one-line notation, putting on a line the ordered sequence $(\pi(1), \ldots, \pi(n))$. A *permutation graph* is the intersection graph of the line segments that connect two parallel lines in the Euclidean plane, one representing the ordered sequence $(1, \ldots, n)$ and one

$(\pi(1), \ldots, \pi(n))$; the i-th segment connects point i on the first line to point $\pi(i)$ on the second line. Equivalently, a permutation graph is a graph for which there is a node for each $1, 2, 3, ...n$ and an edge between i and j if and only if $i < j$ and $\pi(i) > \pi(j)$.

Let us now analyze the permutation diagram of a split permutation graph, $G = (K, I, E_\pi)$: the line segments corresponding to nodes in K have as endpoints on the second line the points of the longest decreasing subsequence in π and pairwise intersecting, while the line segments corresponding to the nodes in I are all disjoint. In Figure 3 a split permutation graph and its relative permutation diagram are depicted.

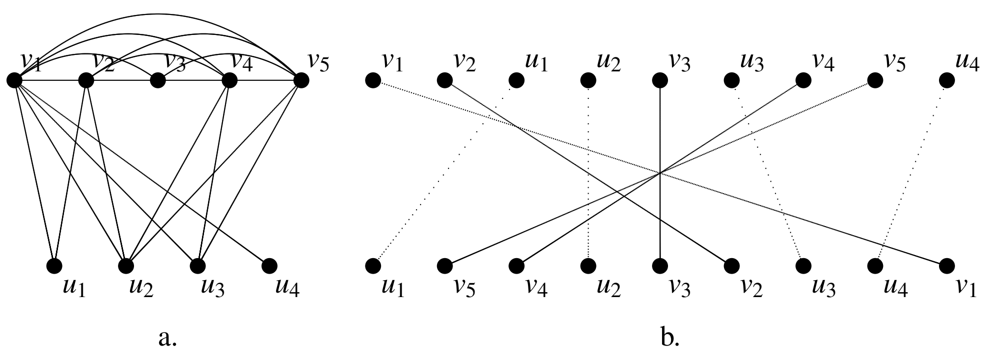

Figure 3: a. A split permutation graph G; b. its permutation diagram.

The class of split permutation graphs is precisely the class of split graphs having Dilworth number at most two. From the other side, the class of split permutation graphs coincides with the intersection between interval and a co-interval graphs [5]. Interval graphs are known to be in LPG [3], and from Proposition 1 it follows that the intersection between interval and co-interval graphs is inside $LPG \cap mLPG$. Nevertheless, if a split permutation graph is given by means of its permutation diagram, it is not clear how to use the results on graphs with Dilworth number two or on interval graphs to construct the tree that witnesses that it is in $LPG \cap mLPG$. In this section we exploit the permutation diagram of split permutation graphs to produce the edge-weighted trees and the values d_{min} and d_{max} witnessing that these graphs are in $LPG \cap mLPG$. To this aim, we first recall the construction that proves that any interval graph is LPG, and then we present two novel transformations that, starting from the permutation diagram of a split permutation graph, produce the interval intersection model of the same graph and the permutation diagram of its complement.

Construction for Interval Graphs. [3] Given an interval graph G by means of its interval model, it is possible to construct a caterpillar C and an edge-weight function w such that $G = LPG(C, w, 1)$ as follows. For each interval I_v, it is not restrictive to assume that $|I_v| \leq 1$ (if not, it is easy to normalize these lengths) and let m_v be its midpoint; intervals are ordered so that $1 \leq i < j \leq n$ if $m_{v_i} < m_{v_j}$. The spine of C is the path $p_1, \ldots, p_n$ and the leaf corresponding to node v_i of G is attached as unique child of p_i. The edge-weight function is defined as follows: $w(p_i, p_{i+1}) = m_{v_{i+1}} - m_{v_i}$ for all $i = 1, \ldots n-1$; $w(p_i, v_i) = \frac{1-|I_{v_i}|}{2}$ for all $i = 1, \ldots, n$.

Transformation 1. *Given a split permutation graph $G = (K, I, E_\pi)$ by means of its permutation diagram, it is possible to deduce its interval intersection model in linear time with respect to the number of nodes of G.*

Proof. Remind that the class of split permutation graphs coincides with the intersection between Dilworth two graphs and split graphs, so it is possible to compute the two chains of G, C_1 and C_2. If a node v is in the neighborhood of a node u in C_i, $i = 1, 2$, then v is also in the neighborhood of all the nodes following u in C_i, by definition of chain. So, let us consider the only nodes involved in the stable set I as partitioned according to the two chains, and ordered according to the inclusion of their neighborhood: $s^1_{i_1}, \ldots, s^1_{i_{d1}}$ and $s^2_{j_1}, \ldots, s^2_{j_{d2}}$. We construct the intervals corresponding to the nodes in I as follows:

- if $s^1_{i_k}$, $k = 1, \ldots, d_1$ is the r-th node in chain C_1, let its corresponding interval a point at x-coordinate equal to r;
- if $s^2_{j_k}$, $k = 1, \ldots, d_2$ is the r-th node in chain C_2, let its corresponding interval a point at x-coordinate equal to $d_1 + 1 + r$;

Let us now consider one by one, in any order, all the nodes in the clique set K and construct the corresponding intervals as follows:

- if c_i is in the neighborhood of the r-th node of C_1 but not of the $(r-1)$-th, let its corresponding interval have its left end-point at x-coordinate equal to r; if c_i is not in the neighborhood of any node of C_1, then let its corresponding interval have its left end-point at x-coordinate equal to $d_1 + 1$;
- if c_i is in the neighborhood of the r-th node of C_2 but not of the $(r-1)$-th, let its corresponding interval have its right end-point at x-coordinate equal to $d_1 + 1 + r$; if c_i is not in the neighborhood of any node of C_2, let its corresponding interval have its right end-point at x-coordinate equal to $d_1 + 1$.

It is easy to see that all the intervals corresponding to nodes in the clique intersect at least at x-coordinate $d_1 + 1$. Furthermore, the point corresponding to the r-th stable node of C_i, $i = 1, 2$ intersect all and only segments corresponding to the nodes in its neighborhood. So the correctness of this construction follows.

The proof is concluded observing that this construction can be performed in a time that is proportional to the number of nodes of G. □

Observe that since the partition into two chains may be not unique, so it is the interval intersection model.

Example. Let us consider the split permutation graph in Figure 3.a. One of the possible partitions of its nodes into two chains is: $C_1 = \{v_1, v_2, v_3, u_2, u_1, u_4\}$ and $C_2 = \{v_4, v_5, u_3\}$. So, following the notation above, $s_1^1 = u_4$, $s_2^1 = u_1$, $s_3^1 = u_2$ and $s_1^2 = u_3$ and their intervals are points at x-coordinates 1, 2, 3 and 5, respectively. The intervals corresponding to nodes in the clique are positioned as in Figure 4, so completing the interval intersection model of the graph.

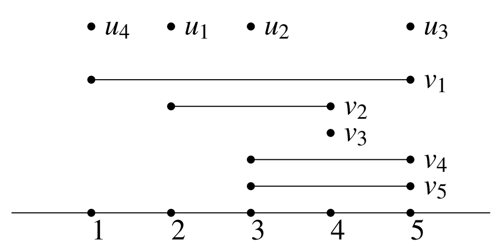

Figure 4: The interval model of the graph depicted in Figure 3.a.

Transformation 2. *Given a split permutation graph $G = (K, I, E_\pi)$ by means of its permutation diagram, it is possible to deduce the permutation diagram of its complement graph $G^C = (I, K, E_\pi^C)$ in linear time with respect to in the number of nodes of G.*

Proof. In order to define the permutation diagram of G^C, observe that the line segments corresponding to clique nodes in G correspond to stable nodes in G^C, so they do not have to intersect anymore, while the line segments corresponding to the stable nodes of G correspond to the clique nodes in G^C and so they must intersect each others. This situation can be obtained just reversing the order of the sequence $(\pi(1), \ldots, \pi(n))$ on its line of the permutation diagram. This new permutation diagram characterizes G^C because en edge (i, j) is in G if and only if i precedes j and $\pi(j)$ precedes $\pi(i)$ in the permutation diagram, but this happens if and only if $\pi(i)$ precedes $\pi(j)$ in the newly computed permutation diagram, that is equivalent to a non-edge in G^C. □

Theorem 2. *Let $G = (K, I, E_\pi)$ be a split permutation graph. A caterpillar C with edge-weight function w and a value d_{min} such that $G = mLPG(C, w, d_{min})$ can be found in polynomial time.*

Proof. Starting from the permutation diagram of G, we deduce its interval intersection model by means of Transformation 1. The proof is completed applying the Construction for Interval Graphs of the edge-weighted caterpillar C and d_{max}. □

Theorem 3. *Let $G = (K, I, E_\pi)$ be a split permutation graph. A caterpillar C with edge-weight function w and a value d_{max} such that $G = LPG(C, w, d_{max})$ can be found in polynomial time.*

Proof. Starting from the permutation intersection model of G, we deduce the permutation diagram of G^C by means of Transformation 2. Since even G^C is an interval graph, we can run on G^C Transformation 1 to deduce its interval diagram. We obtain the claim by applying the Construction for Interval Graphs to deduce the edge-weighted caterpillar C and d_{max}. □

5. Conclusions and Open Problems

In this paper, we have dealt with PCGs and their subgraphs LPGs and mLPGs. It is known that graphs with Dilworth number 1 (threshold graphs) are in $LPG \cap mLPG$; we progressed in the characterization of this intersection proving that also graphs with Dilworth number two belong to it.

From this result, it naturally arises the question if it is possible to completely characterize the class $LPG \cap mLPG$, listing all the graphs belonging to it, besides Dilworth at most two graphs.

Concerning the graphs with higher Dilworth number, in [9] it has been proven that LPGs and mLPGs of trees obtained connecting the centers of k stars with a path are Dilworth k graphs, but the opposite is not necessarily true for $k \geq 3$.

It follows that other classes of graphs belonging to $LPG \cap mLPG$, if they exist, have to be searched among classes not characterized by their chains.

In this paper we also focus on split permutation graphs, i.e. the subclass of graphs with Dilworth number two that are also split graphs. Since these graphs coincide with the intersection between interval and a co-interval graphs, we show how to deduce the corresponding interval model from the permutation one. The relations between permutations and intervals provide a specific way to compute the tree and the values d_{min} and d_{max}. This transformation, that starts from the permutation diagram of a split permutation graph producing the interval intersection model of the same graph and the permutation diagram of its complement, appears inherently interesting and we catch sight of new applications for it.

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