Proof Systems that Take Advice

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One of the starting points of propositional proof complexity is the seminal paper by Cook and Reckhow [5], where they defined propositional proof systems as poly-time computable functions which have all propositional tautologies as their range. Motivated by provability consequences in bounded arithmetic, Cook and Krajíček [4] have recently started the investigation of proof systems which are computed by poly-time functions using advice. While this yields a more powerful model, it is also less directly applicable in practice.

In this talk we describe recent results on this new model of proof systems. In particular, we investigate the following questions:

1. Do there exist optimal or p-optimal proof systems with advice?
2. Do there exist polynomially bounded proof systems with advice?
3. Does the usage of advice shorten propositional proofs?

While the first question receives an affirmative answer [4, 2], we obtain different characterizations for the second one, depending on the complexity of the underlying language and the amount and type of the advice used by the proof system [1]. To approach the third question, we compare proof systems with advice with both classical advice-free systems and proof systems using an easy oracle [3].

Results described in the talk are joint work with Johannes Köbler and Sebastian Müller [1–3].

References

1. O. Beyersdorff, J. Köbler, and S. Müller. Nondeterministic instance complexity and proof systems with advice. In *Proc. 3rd International Conference on Language and Automata Theory and Applications*, volume 5457 of *Lecture Notes in Computer Science*, pages 164 – 175. Springer-Verlag, Berlin Heidelberg, 2009.
2. O. Beyersdorff and S. Müller. A tight Karp-Lipton collapse result in bounded arithmetic. In *Proc. 17th Annual Conference on Computer Science Logic*, volume 5213 of *Lecture Notes in Computer Science*, pages 199 – 214. Springer-Verlag, Berlin Heidelberg, 2008.
3. O. Beyersdorff and S. Müller. Does advice help to prove propositional tautologies? To appear in *Proc. 12th International Conference on Theory and Applications of Satisfiability Testing*, Lecture Notes in Computer Science. Springer-Verlag, Berlin Heidelberg, 2009.
4. S. A. Cook and J. Krajíček. Consequences of the provability of NP $\subseteq$ P/poly. *The Journal of Symbolic Logic*, 72(4):1353–1371, 2007.
5. S. A. Cook and R. A. Reckhow. The relative efficiency of propositional proof systems. *The Journal of Symbolic Logic*, 44(1):36–50, 1979.