Unprovability in Ramsey theory

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The talk will consist of three sections, all related to Ramsey theory and unprovability.

Finitary versions of the infinitary Pigeonhole Principle and the infinitary Ramsey theorem

Joint work with Andreas Weiermann

Firstly, we show how iterations of a finite version of the Pigeonhole Principle preserve the strength of the infinitary Pigeonhole Principle, i.e. $\forall c\, \mathrm{RT}^1_c$.

Secondly, we present a miniaturisation of the full infinite Ramsey theorem ($\forall n\, \mathrm{RT}^n$), using the notion of α-largeness, to reach the strength of $\forall n\, \mathrm{RT}^n$. In both cases we use J. Paris's notion of n-dense sets of natural numbers.

Model-theoretic analysis of Π^1_1-CA$_0$ [and the first-order Ramsey Principle equivalent 1-Con(Π^1_1-CA$_0$)]

Joint work with Andrey Bovykin

We discover a model-theoretic treatment and analysis of initial segments that satisfy Π^1_1-CA$_0$, in the style of Paris and Kirby. We obtain an infinitary statement equivalent to Π^1_1-CA$_0$ and the ultimate result (the Matryoshka Principle), a Π^0_2 statement equivalent to 1-Con(Π^1_1-CA$_0$).

New Nash-Williams-style theorems and the question of their strength

Joint work with Andrey Bovykin

Step by step we generalise a theorem of Nash-Williams ("All thin families of sets are Ramsey"). We study generalisations to labelled trees, labelled graphs and general labelled structures with relations on it. Below is the version for labelled trees.

Definition 1. *A labelled tree is a tree T with an assignment $f : V(T) \to \mathbb{N}$ such that for all vertices u and v, $u \prec_T v$ implies $f(u) < f(v)$ and each label appears finitely many times. If T_1 and T_2 are labelled trees, then T_1 is a direct subtree of T_2 ($T_1 \sqsubseteq T_2$) if there is a label-preserving embedding $g : T_1 \to T_2$ such that there is no element of T_1 above any element of $T_2 \setminus T_1$. A family $\mathcal{F}$ of finite labelled trees is called thin if $T_1 \not\sqsubseteq T_2$ for every pair T_1, T_2 of distinct members of $\mathcal{F}$.*

Theorem 1. *For any infinite labelled tree T, for every thin family $\mathcal{F}$ of finite labelled trees and any partition $\mathcal{F} = \mathcal{F}_0 \cup \mathcal{F}_1$, there is an infinite labelled tree $S \subseteq T$ such that finite subsets of S are in at most one member of the partition.*

*Aspirant Fonds Wetenschappelijk Onderzoek - Flanders