Rectangle-Free Coloring of Grids

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The following is known.

Theorem 1. *For every $c \geq 1$ there exists natural number $G = G(c)$ such that, for every c-coloring of $G \times G$ there exists a square all of whose corners are the same color.*

There are several proofs of this theorem; however, in all of them the function $G(c)$ is bounded by a very fast growing function.

What if instead of trying to get a square we only want to get a rectangle? How are the bounds then? That is the subject of this talk.

We call a grid c-colorable if there is a way to c-color it and not get any monochromatic rectangles. In this talk we completely characterize which grids are 2-colorable and which grids are 3-colorable. We also make some progress on 4-colorable. The proofs are a nice blend of combinatorics and computer science.

Results described in the talk are joint work with Stephen Fenner, Charles Glover, Semmy Purewal.