Ramsey's theorem for pairs and provably recursive functions

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Abstract

We address the strength of Ramsey's theorem for pairs (RT^2_2) over a weak base theory from the perspective of 'proof mining'. Let RT^{2-}_2 denote Ramsey's theorem for pairs where the coloring is given by an explicit term involving only numeric variables. We add this principle to a weak base theory that includes weak König's lemma and a substantial amount of Σ^0_1-induction (enough to prove the totality of all primitive recursive functions but not of all primitive recursive functionals). In the resulting theory we show using the elimination of monotone Skolem functions (see [1]) the extractability of primitive recursive programs and uniform bounds from proofs of $\forall\exists$-theorems.

We also comment on ongoing work in a more general setting.

References

[1] Ulrich Kohlenbach, *Elimination of Skolem functions for monotone formulas in analysis*, ***Archive for Mathematical Logic***, vol. 37 (1998) pp. 363-390.

[2] Alexander Kreuzer, Ulrich Kohlenbach, *Ramsey's Theorem for pairs and provably recursive functions*, to appear in ***Notre Dame Journal for Formal Logic***.