Exact Ramsey theory: What's the point?!

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In [1] it was established (computationally) that $\mathrm{vdw}_2(6,6) = 1132$ holds, that is, whenever $\{1, \ldots, n\}$ for $n \geq 1132$ is partitioned into two sets, then at least one of these parts must contain an arithmetic progression of size 6 (while this is not true for smaller n). It took a lot of effort to arrive at that knowledge, and there is something "special" about such results — but what is it good for? And, more generally, what is "exact Ramsey theory", which is about the computation of such (concrete) numbers, good for?

The starting point for us is the result of initial experimental work [2] which shows that problems from the field of Ramsey theory offer very good benchmarks to improve SAT solving: On the one hand the problem instances exhibit "structure" in great variety, while on the other hand, similar to random problems, a decent parameterisation is possible; last, but not least, the SAT translations are very natural, and SAT solvers seem to perform always best.

This talk now reports on systematically organising the exploration of the SAT-Ramsey-connection — towards better SAT solving (especially for hard unsatisfiable problems), and investigating combinatorial structures of a wide variety of problems from Ramsey theory. The context is the open-source research environment `OKlibrary` (`http://www.ok-sat-library.org/`).

References

[1] Michal Kouril and Jerome L. Paul. The van der Waerden number $W(2,6)$ is 1132. *Experimental Mathematics*, 17(1):53–61, 2008.

[2] Oliver Kullmann. Exact Ramsey theory in the integers: van der Waerden numbers, Green-Tao numbers, and SAT. Technical Report CSR 2-2009, Swansea University, Computer Science Report Series (`http://www.swan.ac.uk/compsci/research/reports/2009/index.html`), October 2009.