Zero-sums in finite abelian groups

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Let $x_1, \dots, x_n$ be elements in a finite abelian group G. We say that this sequence contains a zero sum, if we can find indices $i_1, \dots, i_k$, such that $x_{i_1} + \dots + x_{i_k} = 0$. Motivated by applications in algebraic number theory, Davenport posed the following problem: Given a group G, determine the least n, such that every sequence of length n contains a zero sum. This problem and related questions got little attention for a long time, however, recent years have seen rapid developments. In this talk I will present the most common methods applied to this and related problems. In the first part I will present algebraic methods, where an auxiliary object, often a polynomial or an element in the group algebra, is constructed, which under certain assumptions on the sequence has properties contradicting general theorems, e.g. the combinatorial Nullstellensatz. In the second part I will talk about analytic methods. Here, one tries to find lower bounds for the number of elements representable as subset sums. Different from algebraic methods here we often have to resort to case distinctions and brute force methods including computer searchs.