From Ramsey to Ehrenfeucht: a reduction between games

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A Ramsey game $AVOID(G, F)$ is played by two players, RED and BLUE, who alternately color edges of a graph G. RED is first to move. A player loses once he creates a monochromatic copy of a graph F. $AVOID(K_6, K_3)$ is well known under the name of SIM. While a winning strategy in SIM belongs to BLUE, not much is known in the general case. For example, it is an open question who of the players has a winning strategy in $AVOID(K_{18}, K_4)$.

We investigate a *mirror strategy* for BLUE on G allowing him not to lose $AVOID(G, F)$ for any F: BLUE just keeps the red and the blue subgraphs of G isomorphic. Let $L(G)$ denote the maximum number of rounds in which BLUE succeeds in doing so independently of RED's strategy. No mirror strategy is available on complete graphs, where we have $L(K_n) \leq 6$. On the other hand, such a strategy can be sometimes very efficient. For example, $L(C_n) = n/2$ for even n, where C_n denotes the cycle of length n. If n is odd, we prove that

$$\Omega(\log n) \leq L(C_n) \leq O(\log^2 n).$$

The proof of the lower bound is based on a connection to the Ehrenfeucht game. We will discuss nonconstructive and constructive versions of this argument. The former proves only the existence of an appropriate mirror strategy, while the latter provides such a strategy explicitly.

Joint work with Frank Harary and Wolfgang Slany.