The first order definability of finite graphs

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We consider the first order theory of graphs in the laconic language containing relation symbols for vertex adjacency and equality. The *logical depth* $D(G)$ of a graph G is equal to the minimum quantifier depth of a sentence defining G up to isomorphism. We will survey known estimates for this and related parameters of a graph and discuss their relations to other research areas. For example:

1. 3-connected planar graphs have logarithmic logical depth in a finite variable logic. This result on the descriptive complexity translates into a result on the computational complexity: Isomorphism of planar graphs can be tested in NC.

2. The logical depth of a random graph is logarithmic. This bound is obtained with high precision and can be used for formulating a quantitative version of the Glebskii-et-al.-Fagin 0-1 law.

3. Let $D_0(G)$ denote the variant of $D(G)$ in the logic with no quantifier alternation. We have $D(G) \leq D_0(G)$ and the gap between the two parameters can be super-recursive. This gives us a link to the work of Frank Plumpton Ramsey in logic, where his combinatorial theorem appeared as a by-product.

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