NONSTANDARD METHODS IN COMBINATORICS OF NUMBERS: A FEW EXAMPLES

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By using the methods of *nonstandard analysis*, R. Jin [3] showed an interesting property about sums of sets of integers, namely that $A+B=\{a+b \mid a\in A,\, b\in B\}$ is piecewise syndetic when A and B have positive Banach density. (Roughly speaking, a set of integers A is *piecewise syndetic* if it has bounded gaps on arbitrarily large intervals. The *Banach density* is a refinement of the upper asymptotic density.)

That result raised the attention of several researchers, and has been subsequently improved by V.Bergelson, H.Furstenberg and B.Weiss who showed that $A+B$ is in fact piecewise Bohr (see [2]). Recently, M.Beiglböck has found a really nice ultrafilter proof of Jin's theorem (see [1]). Starting from that initial work, R. Jin applied again nonstandard methods to attack density problems in additive number theory producing a series of interesting results (see *e.g.* [4,5]).

The goal of my talk is to present a few examples in combinatorial number theory that suggest nonstandard analysis as a useful tool in this area of research.

The first example is about intersection properties of sets of differences $A-A$ of natural numbers which only depend on their relative density.

A second example is an ultrafilter proof of the partition regularity of injective solutions of linear diophantine equations (the point here is that – in a nonstandard setting – ultrafilters can be identified with points, namely hypernatural numbers). This technique can also be applied in the study of some non-linear equations.

The third example is a property that improves on Jin's theorem. The nonstandard proof we give uses familiar elementary arguments applied in the nonstandard setting.

Bibliography.

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