Pandelis Dodos Density Ramsey Theory

We shall review some recent advances in Ramsey Theory for trees focusing in particular on the Halpern-Läuchli Theorem.

The Halpern-Läuchli Theorem, discovered in 1966, is a deep pigeon-hole principle for trees and concerns partitions of the level product of a finite sequences of uniquely rooted, finitely branching trees without maximal nodes. It has been the main tool for the development of Ramsey Theory for trees, a rich area of Combinatorics with significant applications, most notably in the Geometry of Banach spaces.

A density version of the Halpern-Läuchli Theorem was conjectured by R. Laver in the late 1960s and obtained recently (2010-2011). We shall discuss the infinite version of the density Halpern-Läuchli Theorem, but we will mainly focus on the finite version and the corresponding bounds obtained. The proof is based on a density increment strategy, a powerful and fruitful method pioneered by K. F. Roth, and uses, in an essential way, a probability measure for trees discovered by H. Furstenberg and B. Weiss (2003).

(This is joint work with V. Kanellopoulos, N. Karagiannis and K. Tyros.)