Alexander Kreuzer

Ramsey's theorem for pairs and program extraction

Let Ramsey's theorem for pairs (RT^2_2) be the statement that every coloring of unordered pairs of $\mathbb{N}$ using 2 colors admits an infinite homogeneous set. We present a method for program extraction from proofs that use RT^2_2. (These programs are provably total relative to Σ^0_2-induction.)

Further, we discuss the chain antichain principle (CAC). This principle states that every infinite partial order posses an infinite chain or an infinite antichain. We present a method to extract primitive recursive programs from proofs that use CAC. This reproves the recent result of Chong, Slaman and Yang that CAC does not imply Σ^0_2-induction. Our result even shows that CAC together with WKL does not imply Σ^0_2-induction.

The principle CAC implies the statement that every sequence of real numbers contains a monotone subsequence. Therefore it also implies the variant of the Bolzano-Weierstrass principle, which states that every bounded sequence of real numbers contains a monotone, slowly converging subsequence. By slowly converging we mean that the sequence converges but we do not require a explicitly given rate of convergence (which is the case in the formalization of the Bolzano- Weierstrass principle used in reverse mathematics). Hence the program extraction for CAC has also possible uses in proofs of statements from Analysis.