Massimo Lauria
The proof complexity of Paris-Harrington tautologies

We initiate the study of the proof complexity of propositional encoding of (weak cases of) concrete independence results. We study the proof complexity of Paris-Harrington's Large Ramsey Theorem for bicolorings of graphs.

We prove a non-trivial conditional lower bound in Resolution and a quasi-polynomial upper bound in bounded-depth Frege. The lower bound is conditional on a hardness assumption for a Weak (quasi-polynomial) Pigeonhole Principle in $\mathsf{Res}(2)$. The proof technique of the lower bound extends the idea of using a combinatorial principle to blow-up a counterexample for another combinatorial principle beyond the threshold of inconsistency. A strong link with the proof complexity of an unbalanced Ramsey principle for triangles is established. This is obtained by applying some highly non-trivial constructions due to Erdős and Mills.

(Joint work with Lorenzo Carlucci and Nicola Galesi.)