

Michael Rathjen*Long Proofs*

Gödel showed that a system of higher order S can prove theorems that a system of lower order T cannot. He also showed that there are theorems F that both S and T can prove but a proof of F in S is much shorter than the shortest proof of F in T . In proof theory there are many results where a system S is reduced to another system T , i.e. it is shown that S is conservative over T with respect to Π_2^0 or arithmetic theorems. Another important technique of proof theory is concerned with extracting numerical bounds from Σ_1^0 theorems. The two techniques can be combined to show that there are specific Σ_1^0 theorems springing forth from wqo theory that have short proofs in a system S but only extremely long proofs in a system T . To arrive at that conclusion, however, requires an investigation of how infinitary proof theory with its use of infinitary proof calculi yields finitistic conservativity results.