

Weak Consequences of Ramsey's theorem for pairs and possible generalizations. Richard A. Shore, Cornell University

We will discuss some consequences of Ramsey's theorem for pairs and two colorings (RT_2^2) that are provably weaker from the viewpoints of both reverse mathematics and Turing computability. In particular, we consider Dilworth's theorem (Chains or AntiChains in partial orders, CAC) and the Erdős-Szekeres theorem (Ascending or Descending Chains in linear orders, ADS) which we prove are weaker than RT_2^2 . Our analysis leads to versions of RT_n^2 where the n -colorings ($n = 2, 3$) of pairs are required to have some type of transitivity properties. These turn out to provide equivalents (in both senses) to CAC and ADS. This is joint work with Denis Hirschfeldt. These "transitive" n -colorings have many possible generalizations to k -tuples for $k > 2$ viewed geometrically or in other ways. With François Dorais, we have some fragmentary results on the strength of some of these generalizations and connections to other variants of Ramsey's theorem. We want to know if these are known types of Ramsey theorems and what are the "right" generalizations to study.