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Unwinding Infinitary Arguments in Combinatorics

Many theorems in combinatorics which "ought" to have computable bounds also have very pleasant proofs which don't appear to give any bounds at all. The functional interpretation has been a powerful method for translating these non-effective proofs into effective ones. Here we will be concerned with combinatorial arguments where the conclusion is that an infinite set exists—prototypical examples are Hindman's Theorem from Ramsey Theory and the Higman's and Nash-Williams Theorems of wqo theory. The natural analog to numeric bounds in this setting are bounds on the complexity of the infinite set produced, one of the main questions of reverse mathematics. We will explain how these proofs can be non-effective, and how an adaptation of the functional interpretation can be used to extract effective bounds on complexity.