

Some Fraïssé
Classes of
Finite Integral
Metric Spaces

Gregory
Cherlin

Metrically Ho-
mogeneous
Graphs

Finite Distance
Transitive Graphs

Homogeneous
Graphs

Homogeneous
Metric Spaces

A Catalog

Special Cases

Generic Cases

Proofs

Conclusion

Some Fraïssé Classes of Finite Integral Metric Spaces

Gregory Cherlin



Bertinoro, May 27

- 1 **Metrically Homogeneous Graphs**
 - Finite Distance Transitive Graphs
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The Classification Problem

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Γ connected, with graph metric d .

Γ is **metrically homogeneous** if the metric space (Γ, d) is (ultra)homogeneous.

(Cameron 1998) Classify the countable metrically homogeneous graphs.

Contexts: infinite distance transitive graphs, homogeneous graphs, homogeneous metric spaces

Finite Distance Transitive Graphs

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distance transitivity = metric homogeneity for pairs

Smith's Theorem:

- Imprimitive case: Bipartite or Antipodal (or a cycle)
Antipodal: maximal distance δ
- Reduction to the primitive case (halving, folding)

Classification of Homogeneous Graphs

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Metrically homogeneous diameter ≤ 2 = Homogeneous.
(The metric *is* the graph)

Fraïssé Constructions: Henson graphs H_n, H_n^c

Lachlan-Woodrow 1980 The homogeneous graphs are

- $m \cdot K_n$ and its complement;
- The pentagon and the line graph of $K_{3,3}$ (3×3 grid)
- The Henson graphs and their complements (including the Rado graph)

Method: Induction on Amalgamation Classes

Claim: If \mathcal{A} is an amalgamation class of finite graphs containing all graphs of order 3, I_∞ , and K_n , then \mathcal{A} contains every K_{n+1} -free graph.

Proof by induction on the order $|A|$ where A is K_{n+1} -free
This doesn't work directly, but a stronger statement can be proved by induction.

Induction via Amalgamation

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\mathcal{A}' is the set of finite graphs G such that any 1-point extension of G lies in \mathcal{A} .

Inductive claim: Every finite graph belongs to \mathcal{A}' .

Not making much progress yet, but . . .

1-complete: complete. 0-complete: co-complete.

\mathcal{A}^p is the set of finite graphs G such that any finite p -complete graph extension of G belongs to \mathcal{A} .

$$\mathcal{A}^p \subseteq \mathcal{A}'$$

\mathcal{A}^p is an amalgamation class

Target: The generators of \mathcal{A} all lie in one \mathcal{A}^p , for some p .

Lachlan's Ramsey Argument

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How to get into \mathcal{A}^p :

1-point extensions of a large direct sum $\bigoplus A_i$
 \implies
 p -extensions of one of the A_i .

If A_i is itself a direct sum of generators, we get a fixed value of p .

First used for tournaments: Lachlan 1984, cf. Cherlin 1988

Homogeneous Metric Spaces

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Rational-valued Urysohn space.

\mathbb{Z} -valued Urysohn space is a metrically homogeneous space.

Or $\mathbb{Z} \cap [0, \delta]$ -valued.

S-valued: Van Thé AMS Memoir 2010

A metrically homogeneous graph of diameter δ is:

A \mathbb{Z} -valued homogeneous metric space with bound δ , and all triangles $(1, i, i + 1)$ allowed (connectivity).

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- Diameter ≤ 2 (Lachlan/Woodrow 1980)
- Locally finite (Cameron, Macpherson)
- Γ_1 -exceptional
- Imprimitive (Smith's Theorem)

The Locally Finite Case

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Finite of diameter at least 3 and vertex degree at least 3:
Antipodal double covers of certain finite homogeneous
graphs (Cameron 1980)

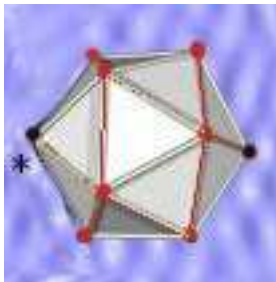


Figure: Antipodal Double cover of C_5

Infinite, Locally Finite: Tree-like $T_{r,s}$ (Macpherson 1982)
Construction:

The graphs $T_{r,s}$

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The trees $T(r, s)$: Alternately r -branching and s -branching. Bipartite, metrically homogeneous if the two halves of the partition are kept fixed.

The graph obtained by “halving” on the r -branching side is $T_{r,s}$.

Each vertex lies at the center of a bouquet of r s -cliques.

Another point of view: the graph on the neighbors of a fixed vertex:

$$\Gamma_1 : r \cdot K_{s-1}.$$

From this point of view, we may also take r or s to be infinite!

$\Gamma_i = \Gamma_i(v)$: Distance i , with the induced metric.

Remark

If distance 1 occurs, then the connected components of Γ_i are metrically homogeneous.

In particular Γ_1 is a homogeneous graph.

Exceptional Cases: finite, imprimitive, or H_n^C .

The finite case is Cameron+Macpherson, the imprimitive case leads back to $T_{r,s}$ with r or s infinite, and H_n^C does not occur for $n > 2$ (Cherlin 2011)

In other words, the nonexceptional cases are

- I_∞
- Henson graphs H_n including Rado's graph.

Imprimitive Graphs

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“Smith’s Theorem” (Amato/Macpherson, Cherlin):

Part I: Bipartite or antipodal, and in the antipodal case with classes of order 2 and the metric antipodal law for the pairing:

$$d(x, y') = \delta - d(x, y)$$

Hence no triangles of diameter greater than 2δ :

$$d(x, z) \leq d(x, y') + d(y', z) = 2\delta - d(x, y) - d(x, y)$$

Part II: The bipartite case reduces by halving to a case in which Γ_1 is the Rado graph.

On the other hand, **the antipodal case does not reduce:** while distance transitivity is inherited after “folding,” metric homogeneity is not.

There is also a bipartite antipodal case.

Some Amalgamation Classes

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Within \mathcal{A}^δ : finite integral metric spaces with bound δ :

- $\mathcal{A}_{K,\text{even}}^\delta$: No odd cycles below $2K + 1$.
- $\mathcal{A}_{C,\text{bounded}}^\delta$: Perimeter at most C .
- $(1, \delta)$ -constraints.

The first two classes are given (implicitly) in Komjath/Mekler/Pach 1988 as examples of constraints admitting a **universal graph**, which is constructed by amalgamation.

The last is a generalization of Henson's construction. A $(1, \delta)$ -space is a space in which only the distances 1 and δ occur (a vacuous condition if $\delta = 2$).

Any set \mathcal{S} of $(1, \delta)$ -constraints may be imposed.

Mixing: $\mathcal{A}_{K,C;\mathcal{S}}^\delta$

Expectations ca. 2008

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- The generic case is $\mathcal{A}_{\Delta, \mathcal{S}}^\delta$ with Δ some set of forbidden triangles ...
- and Δ is a mix of parity constraints K and size constraints C .

Not quite ...

Variations on a theme

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More examples

- $C = (C_0, C_1)$: C_0 controls large even parity, C_1 controls large odd parity

Variations on a theme

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More examples

- $C = (C_0, C_1)$: C_0 controls large even parity, C_1 controls large odd parity
- $K = (K_1, K_2)$: K_1 controls odd cycles at the bottom, K_2 controls odd cycles midrange.
 - (i, j, k) : $P = i + j + k$
 - For P odd, forbid

$$P < 2K_1 + 1$$

$$P > 2K_2 + i$$

Triangle Constraints

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Theorem

If \mathcal{A} is a geodesic amalgamation class of finite integral metric spaces with diameter δ , determined by triangles, then \mathcal{A} is one of the classes

$$\mathcal{A}_{K,C;\delta}^{\delta}$$

with $K = (K_1, K_2)$ and $C = (C_0, C_1)$.

But not all such classes work

Definability in Presburger Arithmetic

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The classes $\mathcal{A}_{K,C}^\delta$ are uniformly definable in Presburger arithmetic from the parameters $K_1, K_2, C_0, C_1, \delta$.

The ***k-amalgamation property*** is amalgamation for diagrams of order at most k .

With constraints of order 3, one expects k -amalgamation for some low k to imply amalgamation. (In the event, $k = 5$.)

Observation

k -amalgamation is a definable property in Presburger arithmetic for the classes $\mathcal{A}_{K,C}^\delta$.

Therefore it should be expressible using inequalities and congruence conditions on linear combinations of the parameters.

Acceptable Parameters

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- $\delta \geq 3$.
- $1 \leq K_1 \leq K_2 \leq \delta$ or $K_1 = \infty$ and $K_2 = 0$;
- $2\delta + 1 \leq C_{\min} < C_{\max} \leq 3\delta + 2$, with one even and one odd.

Conditions for amalgamation (or 5-amalgamation):

Conditions on K, C

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- If $K_1 = \infty$:

$$K_2 = 0, C_1 = 2\delta + 1,$$

- If $K_1 < \infty$ and $C \leq 2\delta + K_1$:

$$C = 2K_1 + 2K_2 + 1; K_1 + K_2 \geq \delta; K_1 + 2K_2 \leq 2\delta - 1$$

$$\text{If } C' > C + 1 \text{ then } K_1 = K_2 \text{ and } 3K_2 = 2\delta - 1.$$

- If $K_1 < \infty$, and $C > 2\delta + K_1$:

$$K_1 + 2K_2 \geq 2\delta - 1 \text{ and } 3K_2 \geq 2\delta.$$

$$\text{If } K_1 + 2K_2 = 2\delta - 1 \text{ then } C \geq 2\delta + K_1 + 2.$$

$$\text{If } C' > C + 1 \text{ then } C \geq 2\delta + K_2.$$

Notes:

$$C = \min(C_0, C_1), C' = \max(C_0, C_1)$$

$C' > C + 1$ means we need both C_0 and C_1 .

Conditions on \mathcal{S}

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- If $K_1 = \infty$:

$$\mathcal{S} \text{ is } \begin{cases} \text{empty} & \text{if } \delta \text{ is odd, or } C_0 \leq 3\delta \\ \text{a set of } \delta\text{-cliques} & \text{if } \delta \text{ is even, } C_0 = 3\delta + 2 \end{cases}$$

- If $K_1 < \infty$ and $C \leq 2\delta + K_1$:

If $K_1 = 1$ then \mathcal{S} is empty.

- If $K_1 < \infty$, and $C > 2\delta + K_1$:

If $K_2 = \delta$ then \mathcal{S} contains no triangle of type $(1, \delta, \delta)$.

If $K_1 = \delta$ then \mathcal{S} is empty.

If $C = 2\delta + 2$, then \mathcal{S} is empty.

Antipodal Variations

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- $\mathcal{A}_a^\delta = \mathcal{A}_{1,\delta-1; 2\delta+2, 2\delta+1; \emptyset}^\delta$ is the set of finite integral metric spaces in which no triangle has perimeter greater than 2δ .
- $\mathcal{A}_{a,n}^\delta$ is the subset of \mathcal{A}_a^δ containing no subspace of the form $I_2^{\delta-1}[K_k, K_\ell]$ with $k + \ell = n$; here $I_2^{\delta-1}$ denotes a pair of vertices at distance $\delta - 1$ and $I_2^{\delta-1}[K_k, K_\ell]$ stands for the corresponding composition, namely a graph of the form $K_k \cup K_\ell$ with K_k, K_ℓ cliques (at distance 1), and $d(x, y) = \delta - 1$ for $x \in K_k, y \in K_\ell$. In particular, with $k = n, \ell = 0$, this means K_n does not occur.

Necessity: Amalgamation diagrams

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Lemma

Let \mathcal{A} be an amalgamation class of diameter δ determined by triangle constraints with associated parameters K_1, K_2, C, C' . Then

$$C > \min(2\delta + K_1, 2K_1 + 2K_2)$$

Necessity: Amalgamation diagrams

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Lemma

Let \mathcal{A} be an amalgamation class of diameter δ determined by triangle constraints with associated parameters K_1, K_2, C, C' . Then

$$C > \min(2\delta + K_1, 2K_1 + 2K_2)$$

We suppose

$$C \leq 2\delta + K_1$$

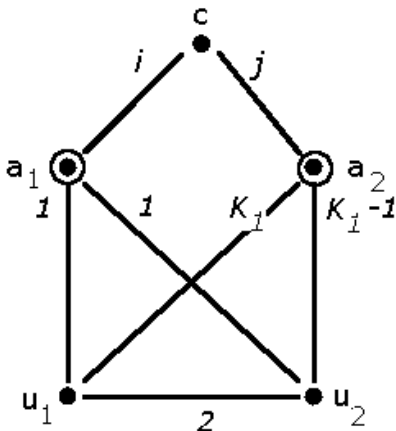
and we show that

$$C > 2K_1 + 2K_2$$

Set $j = \lfloor \frac{C-K_1}{2} \rfloor$, and $i = (C - K_1) - j$. Then $1 < j \leq i \leq \delta$.

$$C > \min(2\delta + K_1, 2K_1 + 2K_2)$$

In the following amalgamation, vertices u_1, u_2 force $d(a_1, a_2) = K_1$ and $|a_1 a_2 c| = C$:



$$d(c, u_1) = d(c, u_2) = i - 1$$

So omit ca_2u_1 or ca_2u_2 , with $P \geq 2K_1 + 1, \dots$

Proofs of amalgamation

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Three amalgamation strategies:

- $d^-(a, b) = \max(d(a, x) - d(a, b))$
- $d^+(a, b) = \inf d(a, x) + d(x, b)$
- $\tilde{d}(a, b) = \inf[C - (d(a, x) + d(a, b))]$

Amalgamation for $\mathcal{A}_{K,C}^\delta$

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- If $C \leq 2\delta + K_1$:
 - If $d^-(a_1, a_2) \geq K_1$ then take $d(a_1, a_2) = d^-(a_1, a_2)$.
Otherwise:
 - If $C' = C + 1$ then:
 - If $d^+(a_1, a_2) \leq K_2$ then take
 $d(a_1, a_2) = \min(d^+(a_1, a_2), \tilde{d}(a_1, a_2))$
 - If $d^-(a_1, a_2) < K_1$ and $K_2 < d^+(a_1, a_2)$ then take
 $d(a_1, a_2) = \tilde{d}(a_1, a_2)$ if $\tilde{d}(a_1, a_2) \leq K_2$ and
 $d(a_1, a_2) = K_1$ otherwise.
 - if $C' > C + 1$ then:
 - If $d^+(a_1, a_2) < K_2$ then take $d(a_1, a_2) = d^+(a_1, a_2)$;
 - If $d^-(a_1, a_2) < K_2 \leq d^+(a_1, a_2)$ then take

$$d(a_1, a_2) = \begin{cases} K_2 - 1 & \text{if there is } v \in A_0 \text{ with} \\ & d(a_1, v) = d(a_2, v) = \delta; \text{ and} \\ K_2 & \text{otherwise} \end{cases}$$

- If $C > 2\delta + K_1$:

Amalgamation for $\mathcal{A}_{K,C}^\delta$

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- If $C \leq 2\delta + K_1$:
- If $C > 2\delta + K_1$:
 - If $d^-(a_1, a_2) > K_1$ then take $d(a_1, a_2) = d^-(a_1, a_2)$;
Otherwise:
 - If $C' = C + 1$ then:
 - If $d^+(a_1, a_2) \leq K_1$ then take
 $d(a_1, a_2) = \min(d^+(a_1, a_2), \tilde{d}(a_1, a_2))$;
 - If $d^+(a_1, a_2) > K_1$ then take

$$d(a_1, a_2) = \begin{cases} K_1 + 1 & \text{if there is } v \in A_0 \text{ with} \\ & d(a_1, v) = d(a_2, v) = \delta, \\ & \text{and } K_1 + 2K_2 = 2\delta - 1 \\ K_1 & \text{otherwise} \end{cases}$$

- If $C' > C + 1$ then:
 - If $d^+(a_1, a_2) < K_2$ then take $d(a_1, a_2) = d^+(a_1, a_2)$;
 - If $d^+(a_1, a_2) \geq K_2$ then take
 $d(a_1, a_2) = \min(K_2, C - 2\delta - 1)$.

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Completeness?

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Good points:

- All cases with exceptional Γ_1
- $\delta \leq 3$, probably (Amato/Cherlin/Macpherson)
- Exact as far as triangle constraints are concerned
- Smith's Theorem

Weak points

- Smith's Theorem
 - Bipartite to be completed inductively
 - Antipodal description may be incomplete
- Induction to Γ_i is not always available

In fact, for **antipodal graphs omitting K_n** , triangles and $(1, \delta)$ -constraints do not suffice.

That class was found on an ad hoc basis.
(It is invisible in diameter 3.)

Toward a classification theorem

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Strategy?

- (Step 0) Prepare diameter 4 and Γ_2 generally? (Prudent)
- (Step 1) Characterize triangles occurring in amalgamation classes
- (Step 2) Show that if the triangle constraints are as expected, then Γ_i has the expected constraints.
- (Step 3) Assuming the first two conditions, characterize Γ .

(Works in diameter 3)

... With Lachlan's Ramsey method in reserve.

Furthermore

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No need to wait for a classification:

- Ramsey theory for these homogeneous metric spaces
- Topological dynamics
- Other aspects of the automorphism group (normal subgroups, subgroups of small index)