Ramsey Theory in Logic, Combinatorics and Complexity

RaTLoCC 2018 - Abstracts of Talks

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Bertinoro International Center for Informatics

Ilario Bonacina (Universitat Politècnica de Catalunya, Barcelona)
Clique is Hard on Average for Regular Resolution

Deciding whether a graph G with n vertices has a k-clique is one of the most basic computational problems on graphs. In this talk we show that certifying k-clique-freeness of Erdős-Rényi random graphs is hard for regular resolution. More precisely we show that, for $k \ll \sqrt{n}$, regular resolution asymptotically almost surely requires length $n^{\Omega(k)}$ to establish that an Erdős-Renyi random graph (with appropriate edge density) does not contain a k-clique. This asymptotically optimal result implies unconditional lower bunds on the running time of several state-of-the-art algorithms used in practice.

This talk is based on on a joint work with A. Atserias, S. De Rezende, M. Lauria, J. Nordström and A. Razborov.

Vasco Brattka (Universität der Bundeswehr München)
Ramsey's Theorem in the Weihrauch Lattice

The Weihrauch lattice has been established as a tool for the classification of uniform computational content and Ramsey's theorem has been studied and classified in this lattice. We present a survey on results that have been obtained and some open questions.

Peter Cholak (University of Notre Dame)
Encodable by thin sets

Let c be a coloring of n-tuples (of ω) by finitely many colors. For l less than the number of colors, a set T is *l-thin* iff c uses at most l colors to color all the n-tuples from T. We say a set S is *$RT^n_{<\infty,l}$-encodable* iff there is a coloring c as above such that every l-thin set computes S. Wang and others 'showed that when l is "large" only the computable sets are $RT^n_{<\infty,l}$-encodable. Dorais, Dzhafarov, Hirst, Mileti, and Shafer showed that the hyperarithmetic sets are $RT^n_{<\infty,l}$-encodable for "small" l. Cholak and Patey showed that the arithmetic sets are $RT^n_{<\infty,l}$-encodable for "medium" l. Of course, what is missing here is the exact definition of small, medium, and large. In the talk we will provide "tight" definitions, at least, for a "few" n.

This is joint work with Ludovic Patey.

Gabriel Conant (University of Notre Dame)
VC-dimension in groups

Abstract: I will discuss recent work on the structure of VC-sets in groups, i.e. subsets whose family of left translates has absolutely bounded VC-dimension. Many tools from model theory and additive combinatorics can be adapted for "locally amenable" VC-sets, leading to stronger structural results than for arbitrary sets in (amenable) groups. These structure results can be applied to questions concerning sumset phenomena for VC-sets in infinite groups, as well as arithmetic regularity for VC-sets in finite groups.

David Conlon (University of Oxford)
Ramsey complete sequences

A sequence of positive integers A is said to be entirely Ramsey complete if, for any two-colouring of A, every positive integer can be written as the sum of distinct elements of A of the same colour. We show that there exists a constant C and an entirely Ramsey complete sequence A such that, for all n, A has at most $C(\log n)^2$ elements up to n. This is best possible up to the constant and solves a problem of Burr and Erdős. We also discuss several related problems stated by the same authors.

Joint work with Jacob Fox.

Natasha Dobrinen (University of Denver):
Ramsey theory of the universal triangle-free graph

It is a central question in the theory of homogeneous relational structures as to which structures have finite big Ramsey degrees. This question, of interest for several decades, has gained recent momentum as it was brought into focus by Kechris, Pestov, and Todorcevic in their 2005 paper, in which they proved a deep correspondence between Ramsey theory of Fraisse limits and topological dynamics. An infinite structure S is said to be homogeneous if any isomorphism between two finitely generated substructures of S can be extended to an automorphism of S. A homogeneous structure S is said to have finite big Ramsey degrees if for each finite substructure A of S, there is a number n, depending on A, such that any coloring of the copies of A in S into finitely many colors can be reduced down to no more than n colors on some substructure S' isomorphic to S. This is interesting not only as a Ramsey property for infinite structures, but also because of its implications for topological dynamics, following recent work of Zucker.

Prior to work of the speaker, finite big Ramsey degrees had been proved for a handful of homogeneous structures: the rationals (Devlin 1979) the Rado graph (Sauer 2006), ultrametric spaces (Nguyen Van Thé 2008), and enriched versions of the rationals and related circular directed graphs (Laflamme, Nguyen Van Thé, and Sauer 2010). According to Nguyen Van Thé, Sauer, and Todorcevic, the lack of tools to represent ultrahomogeneous structures with forbidden configurations, particularly the lack of any analogue of Millikens Ramsey theorem for strong trees applicable to such structures, was a major obstacle towards a better understanding of their infinite partition properties.

The universal triangle-free graph is the simplest homogeneous structure with a forbidden configuration. Prior to my work, Komjath and Rödl had proved in 1986 that vertex coloring have big Ramsey degree 1, and Sauer had proved in 1998 that edge colorings have big Ramsey degree 2. It was a major open problem in the Ramsey theory of homogeneous structures whether or not each finite triangle has a finite big Ramsey degree. Recently, I solved this problem, in the process developing new techniques to represent the universal triangle-free graph via certain trees, and developing the Ramsey theory of these trees to deduce finite bounds for the big Ramsey degrees. Work in progress is the theorem that all Henson graphs, the universal k-clique-free graphs, have finite big Ramsey degrees. The methods developed seem robust enough that correct modifications should likely apply to a large class of homogeneous structures with forbidden configurations.

Pandelis Dodos (National and Kapodistrian University of Athens)
An inverse theorem for stochastic processes indexed by the discrete hypercube

Let A be a nonempty finite set, let n be a positive integer, and let A^n denote the discrete n-dimensional hypercube (that is, A^n is the Cartesian power of n many copies of A). Given a family $\langle D_t : t \in A^n\rangle$ of measurable events in a probability space (a stochastic process), what structural information can be obtained assuming the the events $\langle D_t : t \in A^n\rangle$ are not "behaving" as if they were independent? We shall describe a complete answer to this problem (in a strong quantitative sense) subject to a mild "stationarity" condition. This result has a number of combinatorial consequences, including a new (and the most informative so far) proof of the density Hales–Jewett theorem. We shall discuss these consequences, and well as, several related problems.

This is joint work with Kostas Tyros.

Damir Dzhafarov (University of Connecticut)
Calling a few good combinatorialists

One of the most fruitful programs of research in computability theory over the past thirty years has been the investigation of the logical strength of combinatorial principles, and of Ramsey's theorem especially. I will discuss a central and longstanding open problem in this area known as the $\mathbf{SRT}^2_2$ vs. $\mathbf{COH}$ problem, which asks whether two versions of Ramseys theorem for pairs are equivalent in a certain precise sense. While the problem is computability-theoretic in nature, the main stumbling blocks seem deeply combinatorial. In particular, recent developments have uncovered related questions that can be stated in more purely combinatorial terms. This suggests a collaborative approach between logicians and combinatorialists may be successful in finding a solution.

Bill Gasarch (University of Maryland)
The First Ramseyian Theorem and its Application: The Hilbert Cube Lemma and Hilbert's Irreducibility theorem

The first Ramseyian theorem was the Hilbert Cube Lemma (HCL). The first application of a Ramseyian Theorem was by Hilbert when he applied HCL to prove the Hilbert irreducibility theorem (HIT). HIT is:

If $f(x,y) \in Z[x,y] - Z[x]$ and for almost all $t \in Z$, $f(x,t)$ is reducible, then $f(x,y)$ is reducible.

We present a Hilbert's proof of HIT, though phrased in modern mathematical language (and in English). We emphasize the role HCL plays in the proof.

Joint work with Mark Villaino and Ken Regan

Neil Hindman (Howard University, Washington DC)
Combining extensions of the Hales-Jewett Theorem with Ramsey Theory in other structures

The Hales-Jewett Theorem states that given any finite nonempty set A and any finite coloring of the free semigroup S over the alphabet A there is a *variable word* over A all of whose instances are the same color. This theorem has some extensions involving more than one variable in the variable word. We show that, when combined with a sufficiently well behaved homomorphism, the relevant variable word simultaneously satisfies a Ramsey-Theoretic conclusion in the other structure. As an example we show that if τ is the homomorphism from the set of variable words into the natural numbers which associates to each variable word w the number of occurrences of the variable in w, then given any finite coloring of S and any infinite sequence of natural numbers, there is a variable word w whose instances are monochromatic and $\tau(w)$ is a sum of distinct members of the given sequence.

Our methods rely on the algebraic structure of the Stone-Čech compactification of S and the other semigroups that we consider. We show for example that the set of points of βN whose members are guaranteed to contain $\tau(w)$ in the example above forms a compact subsemigroup of βN containing all of the idempotents.

Jeffry Hirst (Appalachian State University)
Hindmans theorem and ultrafilters

Hindman's theorem is equivalent to the existence of certain ultrafilters on the power set of the natural numbers. In fact, it is provable from the existence of these ultrafilters on countable Boolean subalgebras of the power set. The Galvin-Glazer proof of Hindman's theorem locates one of these ultrafilters using a form of addition on ultrafilters. When restricted to countable Boolean algebras, this addition displays some interesting changes in behavior. This talk will relate this observation to the reverse mathematics and computability theory of Hindman's theorem.

Jan Hubicka (Charles University, Prague)
On existence of Ramsey expansions

A class K of structures is Ramsey if for every A, B in K there exists C in K such that for every 2-coloring of copies of A in C there exists a copy of B in C which is monochromatic. Around Bertinoro meeting in 2011 a question asking whether every amalgamation class can be turned into a Ramsey class by means of finite amount of additional information (a Ramsey expansion) was formulated. We discuss a counter-example to the most general form of this question and analyze its Ramsey properties. We also discuss positive results developed on the quest of answering this question.

This is joint work with David Evans and Jaroslav Nešetril.

Carl Jockusch (University of Illinois at Urbana-Champaign)
The strength and effective content of some restricted forms of Hindman's Theorem

Hindman's Theorem states that if the natural numbers are colored with finitely many colors, there is an infinite set H of natural numbers such that all finite nonempty sums of distinct elements of H have the same color. The strength and effective content of Hindman's Theorem were studied by Blass, Hirst, and Simpson in [1]. Recently there have been several investigations of the strength and effective content of restricted versions of Hindman's Theorem in which the sums considered are of bounded length, or all of the same length. I will survey some of this work and concentrate on the following theorem from [2]: There is a computable two-coloring of the natural numbers such that no infinite Σ^0_2 set H has the property that all sums $a + b$ for a, b distinct elements of H have the same color. The proof of this result uses an effective version of the Lovász Local Lemma due to Rumyantsev and Shen [4]. I will also discuss generalizations of this result and corresponding results in reverse mathematics. This is joint work with Barbara Csima, Damir Dzhafarov, Denis Hirschfeldt, Reed Solomon, and Linda Brown Westrick.

[1] A. R. Blass, J. L. Hirst, and S. G. Simpson, Logical analysis of some theorems of combinatorics and topological dynamics, pp. 125 - 156 in Logic and Combinatorics, volume 65 of Contemporary Mathematics, American Mathematical Society, Providence R. I., 1987.

[2] B. F. Csima, D. D. Dzhafarov, D. R. Hirschfeldt, C. G. Jockusch, D. R. Solomon, and L. B. Westrick, The reverse mathematics of Hindman's Theorem for sums of exactly two elements

[3] D. D. Dzhafarov, C. G. Jockusch, D. R. Solomon, and L. B. Westrick, Effectiveness of Hindman's Theorem for Bounded Sums, pp. 134-142 in Computability and Complexity, Lecture Notes in Computer Science 10010, Springer, 2017.

[4] A. Rumyantsev and A. Shen, Probabilistic constructions of computable objects and a computable version of Lovsz Local Lemma, Fundamenta Informaticae 132 (2014), no. 1, 1-14.

Vladimir Kanovei (Institute for Information Transmission Problems, Moscow)
Canonization on product and iterated perfect and large perfect sets

Some canonization results, related to Borel equivalence relations modulo restriction to various categories of perfect sets, including product and iterated perfect sets, will be presented and commented.

Leszek A. Kołodziejczyk (University of Warsaw)
Ramsey for pairs and proof size

A recent important result of Patey and Yokoyama states that any Π^0_3 statement which is provable from Ramsey's theorem for pairs and two colours is also provable in $\mathbf{RCA}_0$: in other words, $\mathbf{RT}^2_2$ is Π^0_3-conservative over $\mathbf{RCA}_0$ (and thus also over Σ_1-induction). I will talk about some joint work with Wong and Yokoyama in which we show that this conservativity result is "feasible" in the following sense: any proof of a Π^0_3 statement in $\mathbf{RT}^2_2$ can be translated into a proof of the same statement in $\mathbf{RCA}_0$ at the cost of an at most polynomial increase in size.

My plan is to focus on the combinatorial content of our argument, although I will say a little bit about the logical aspects as well. If time permits, I will also explain why an analogous conservativity result for $\mathbf{RT}^2_2$ and $\mathbf{RCA}^*_0$ (that is, $\mathbf{RCA}_0$ without Σ_1-induction) is not feasible in the sense described above.

Lorenzo Luperi Baglini (University of Vienna)
Partition regularity of nonlinear Diophantine equations

In the recent past, several problems regarding the partition regularity of nonlinear configurations have been solved. In this talk, we want to present some general sufficient and necessary conditions for the partition regularity of Diophantine equations, which extend Rado's Theorem and some other classic result by covering large classes of nonlinear equations.

The techniques we use to obtain these conditions are twofold: sufficient conditions are obtained by exploiting algebraic properties in the space of ultrafilters $\beta\mathbb{N}$, grounding on combinatorial properties of positive density sets and IP sets; necessary conditions are obtained by means of some algebraic considerations based on a nonstandard approach to ultrafilters.

Jaroslav Nešetřil (Charles University, Prague)
Ramsey classes and beyond

We survey recent progress on characterization of Ramsey classes, particularly those defined as lifts of classes defined by forbidden substructures.

This is a joint work with D. Evans (London) and J. Hubicka (Prague)

Lionel Nguyen Van Thé (Université d'Aix-Marseille)
Revisiting the Erdős-Rado canonical partition theorem

One of the numerous strengthenings of Ramsey's theorem is due to Erdős and Rado, who analyzed what partition properties can be obtained on m-subsets of the naturals when colorings are not necessarily finite. Large monochromatic sets may not appear in that case, but there is a finite list of behaviors, called "canonical", to which every coloring reduces. The purpose of this talk will be to remind certain not so well-known analogous theorems of the same flavor that were obtained by Prömel in the eighties for various classes of structures (like graphs, hypergraphs, or Boolean algebras), and to show that such theorems can in fact be deduced in the more general setting of Fraïssé classes.

Aaron Robertson (Colgate University)
Ramsey Objects and Delaporte

We investigate the distributions of the number of monochromatic objects over 2-colorings. We know, by the Poisson paradigm, that such distributions tend toward the Poisson distribution; however, this is an asymptotic result. In this talk, we investigate the distributions in the non-asymptotic setting.

Marcin Sabok (McGill University)
The Hrushovski property for hypertournaments and profinite topologies

It is an open problem whether the Hrushovski extension property holds for tournaments. It is equivalent to a problem concerning a profinite topology and a characterization of closed f.g. subgroups in this topology. During the talk I will discuss a generalization of the latter problem to a family of profinite topologies.

Richard Shore (Cornell University)
Producing Elementary Proofs from Ultrafilter Proofs of Combinatorial Theorems: The View from Combinatorics

An increasingly common phenomena in combinatorics is the use of higher order notions, objects and principles to prove combinatorial facts about the natural numbers $\mathbb{N}$ (or some other countable set) and its subsets. One particularly fruitful such notion has been that of (nonprincipal) ultrafilters on $\mathbb{N}$ and it has been applied extensively in proving Ramsey type theorems. We present a method that incorporates a systematic analysis of the classical proofs that exploit algebraic structures on classes of ultrafilters. It produces elementary proofs at relatively low levels of the reverse mathematical/recursion theoretic hierarchies Our results typically show that the theorems are reverse mathematically equivalent to Iterated HIndman's Theorem and that the homogenous sets produced are recursive in the $\omega \cdot m$th Turing jump of the given coloring. (Here both m and the index of the reduction are fixed for all instances of the particular theorem.) Examples include Gowers' Fin_k Theorem and the Infinite Hales-Jewett Theorem.

In this talk we will give an outline of our methods centered on one of the applications from the viewpoint of combinatorics eliminating logical notions almost entirely. That, is we describe a construction of the desired homogenous set which is very elementary. Other than using the standard elementary method of producing solutions to instances of Iterated HIndman's Theorem all other steps are from a specified list of simple (arithmetical or even computable) procedures.

This is joint work with Antonio Montalbán.

Jozef Skokan (London School of Economics)
The k-colour Ramsey number of odd cycles via non-linear optimisation

For a graph G, the k-colour Ramsey number $R_k(G)$ is the least integer N such that every k-colouring of the edges of the complete graph K_N contains a monochromatic copy of G. Let C_n denote the cycle on n vertices. We show that for fixed $k > 2$ and n odd and sufficiently large,

$$R_k(C_n) = 2^{k-1}(n-1) + 1.$$

This generalises a result of Kohayakawa, Simonovits and Skokan and resolves a conjecture of Bondy and Erdős for large n. We also establish a surprising correspondence between extremal k-colourings for this problem and perfect matchings in the hypercube Q_k. This allows us to in fact prove a stability-type generalisation of the above. The proof is analytic in nature, the first step of which is to use the Regularity Lemma to relate this problem in Ramsey theory to one in convex optimisation.

This is joint work with Matthew Jenssen.

Slawomir Solecki (Cornell University):
Ramsey theory in algebraic topological language

We give a new presentation of finite Ramsey theory using simplicial complexes and simplicial maps, which brings Ramsey theory closer to algebraic topology. The main theorem gives an upper bound on Ramsey degrees via the size of images of certain simplicial maps. We also show how infinite Ramsey theory fits into this picture.

Henry Towsner (University of Pennsylvania)
Generalizing VC dimension to higher arity

The notion of bounded VC dimension is a property at the intersection of combinatorics and probability. This family has been discovered repeatedly and studied from various perspectives - for instance, in model theory, theories with bounded VC dimension are known as NIP (the theories which do Not have the Independence Property). One useful property is that graphs with bounded VC dimension are the graphs that can be always be finitely approximated in a random-free way: graphs with bounded VC dimension satisfy a strengthening of Szemeredis Regularity Lemma in which the densities between the pieces of the partition are either close to 0 or close to 1. The generalization of VC dimension to higher arity, known in model theory as k-NIP for various k, has been less well-studied. We summarize some known facts about this generalization, including a new result (joint with Chernikov) showing k-NIP hypergraphs have a similar kind of approximation with only lower order randomness.

Timothy Trujillo (Sam Houston State University)
Hypernatural numbers in ultra-Ramsey theory

We present an alternative formalism to deal with ultrafilters grounded on the use of the infinite hypernatural numbers of nonstandard analysis, which - in a precise sense - replace the use of ultrafilters. We then use the formalism to prove a new infinite-dimensional extension of Ramsey's theorem for ultrafilter trees. We conclude by extending the result to the abstract setting of topological Ramsey spaces.

Anush Tserunyan (University of Illinois)
Independent sets in finite and algebraic hypergraphs

An active line of research in modern combinatorics is extending classical results from the dense setting (e.g. Szemerédi's theorem) to the sparse random setting. These results state that a property of a given "dense" structure is inherited by a randomly chosen "sparse" substructure. A recent breakthrough tool for proving such statements is the Balogh–Morris–Samotij and Saxton–Thomason hypergraph containers method, which bounds the number of independent sets in finite hypergraphs. Jointly with Bernshteyn, Delcourt, and Towsner, we give a new—elementary and nonalgorithmic—proof of the containers theorem for finite hypergraphs. Our proof is inspired by considering hyperfinite hypergraphs in the setting of nonstandard analysis, where there is a notion of dimension capturing the logarithmic rate of growth of finite sets. Applying this intuition in another setting with a notion of dimension, namely, algebraically closed fields, we prove an analogous theorem for "dense" algebraically definable hypergraphs: any Zariski-generic low-dimensional subset of such hypergraphs is itself "dense" (in particular, not independent). The latter is joint with Bernshteyn and Delcourt.

Iddo Tzameret (University of London)
Linear algebra in weak formal theories of arithmetic

I will demonstrate how one can prove and develop basic linear algebra in what is apparently the weakest formal theory of arithmetic possible. This aligns with the direction of research that attempts to bridge the gap between the computational complexity of functions and the logical/computational-complexity of concepts required to prove basic statements about the functions. In our case, we show that to prove basic properties of the determinant function it is enough to reason with concepts of the same complexity class that can compute the determinant function.

Joint work with Stephen Cook.

Douglas Ulrich (University of Maryland)
Distinct Volume Subsets: The Uncountable Case

Erdős proved that for every infinite $X \subseteq \mathbb{R}^d$, there is $Y \subseteq X$ with $|Y| = |X|$, such that all pairs of points from Y have distinct distances, and gave partial results for general a-ary volume. In joint work with William Gasarch, we continue this line of investigation, and search for the strongest possible canonization results for a-ary volume. The main difficulty is for singular cardinals; to handle this case we prove that if T is a complete ω-stable theory, and Δ is a finite set of formulas of T, then whenever X is infinite, there is $Y \subseteq X$ of the same cardinality and an equivalence relation E on Y, with infinitely many classes, each class infinite, such that Y is (Δ, E)-indiscernible. We also consider the definable version of these problems, for example we assume X is perfect (in the topological sense) and we require Y to be perfect. Finally we show that Erdős's theorem requires some use of the axiom of choice.