On Propagation of Phenomena in Interdependent Networks

Hana Khamfroush, Novella Bartolini, Thomas F. La Porta, *Fellow, IEEE*, Ananthram Swami, *Fellow, IEEE*, and Justin Dillman

Abstract—When multiple networks are interconnected because of mutual service interdependence, propagation of phenomena across the networks is likely to occur. Depending on the type of networks and phenomenon, the propagation may be a desired effect, such as the spread of information or consensus in a social network, or an unwanted one, such as the propagation of a virus or a cascade of failures in a communication or service network. In this paper, we propose a general analytic model that captures multiple types of dependency and of interaction among nodes of interdependent networks, that may cause the propagation of phenomena. The above model is used to evaluate the effects of different diffusion models in a wide range of network topologies, including different models of random graphs and real networks. We propose a new centrality metric and compare it to more traditional approaches to assess the impact of individual network nodes in the propagation. We propose guidelines to design networks in which the diffusion is either a desired phenomenon or an unwanted one, and consequently must be fostered or prevented, respectively. We performed extensive simulations to extend our study to large networks and to show the benefits of the proposed design solutions.

Index Terms—Interdependent networks, information diffusion, failure propagation, network design

1 Introduction

Much of today's infrastructure is organized in the form of interdependent networks. Systems of water and food supply, communications, fuel, financial transactions, power generation and transmission are all examples of interdependent networks, where the functionality or performance of one network depends on the other. A failure in one network may cause service degradation or failure in the others. Failures can *cascade* multiple times between two interdependent networks and therefore, result in a catastrophic spread, as described in the work of Parandehgheibi et al. [1].

Nevertheless, failures are not the only phenomenoa that can spread through interdependent networks. A similar process of propagation can be observed in interconnected social networks when information spreads among the nodes of overlapping communities.

There is a considerable amount of literature addressing the problem of modeling information diffusion or cascading failures within a single network or across several interdependent networks. Most of the works on cascading failures, from those of Buldyrev et al. [2], [3], [4], [5], [6] to the more recent works of D'Souza et al. [7], [8], are interested in studying the network fragmentation resulting from the propagation of a failure. They aim at characterizing the conditions under which a phenomenon can spread through a network, though still preserving the existence of a giant working component of connected nodes in the network.

Parallel to this, there is a large body of work which focuses on opinion dynamics, and attempts to characterize the spread of information in a network under different models of interaction among nodes, as in the works of Leskovec et al. [9] and Kleinberg et al. [10], [11], [12].

Some works [5], [13], [14], [15], [16] study the propagation of phenomena through interdependent networks under different scenarios. Only few of them, such as those from Parshani et al. [5] and recently Yagan et al. [13] consider engineering aspects of inter-dependent networks with the purpose of either preventing or hastening cascades in a general scenario.

Unlike previous work, in this paper we aim to provide a framework that generalizes the study of phenomena propagation, by developing a neutral model that allows the study of both unwanted propagation and desired diffusion. We do not make any assumption on the network topology, type of phenomena or inter-connectivity model. The provided framework considers heterogeneous interconnected networks, and heterogeneous nodes that may have different characteristics in terms of their propensity to be involved in the propagation of a phenomenon and to affect other nodes.

We develop a general model that captures the interdependency between two general networks and incorporates an aleatory delay in the occurrence of propagation from node to node. By means of this model, we are able to perform a structural analysis of the interdependent networks, and to study the impact of characteristics such as inter-connectivity patterns and network structure in speeding up or slowing down the propagation of phenomena throughout the networks.

- *H. Khamfroush, T.F. La Porta, and J. Dillman are with the Department of Computer Science and Engineering, Penn State University, State College, PA 16801. E-mail: {hkham, tlp, jnd5215}@cse.psu.edu.*
- *N. Bartolini is with the Computer Science Department, Sapienza University of Rome, Roma 00185, Italy. E-mail: novella@di.uniroma1.it.*
- *A. Swami is with United States Army Research Laboratory, Adelphi, MD 20783. E-mail: ananthram.swami.civ@mail.mil.*

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The purpose of this analysis is to provide guidelines for the design of inter-connectivity schemes that allow one to control the speed and extent of the propagation process.

The contributions of this paper are the following:

- We propose a general threshold-based model for the propagation of phenomena in heterogeneous interdependent networks. This model generalizes the characteristics of other previous approaches.
- We use an absorbing Markov model to investigate the time process of the propagation.
- We perform extensive simulations to study the propagation process over time under different scenarios. We evaluate the impact of different parameters on the speed of propagation, showing that those defining the inter-connectivity play a major role in affecting the propagation process.
- We give design guidelines to make phenomena propagation more controllable. We analyze the impact of the degree of interconnecting nodes on the speed of propagation within networks. We also introduce a new metric of centrality that captures the importance of a node in the propagation of phenomena and propose ways to control the propagation process by altering the characteristics of few high centrality nodes.

2 Related Work

A wide range of studies have been carried out in recent years to predict and investigate the propagation of phenomena across networked systems. Most of these focus on one single topology and/or only investigate the asymptotic behavior of the network. For example, the initial work by Buldyrev et al. [3] considers a one-to-one coupling model between two networks of identical size, where each node in a network depends on only one node in the other. Like in most of the works based on percolation theory, the work of Buldyrev relates the robustness of the network to the size of the giant connected component. The authors study the case of a random attack on some nodes, and show that as long as the fraction of initially failed nodes is below a critical threshold, a significant part of the network remains functional.

A similar problem is tackled by works in the area of bootstrap percolation, such as the work of Chalupa et al. [17] and Baxter et al. [18], where the authors assume that the propagation is ignited by a set of initially affected spreader nodes. In [4], Buldyrev et al. focus on the role of interconnectivity links between two interdependent networks in influencing the speed and extent of propagation, in a simplified scenario where all the interconnecting nodes have the same degree. Other later works tackle more realistic models of inter-connectivity. Parshani et al. [5] study interdependent networks using percolation theory and show that reducing interdependencies between networks below a critical value yields a continuous percolation transition. Stanley et al. [6] generalize previous models by assuming random interdependency between networks.

Other works give more emphasis to robust network design.The work of Schneider et al. [19] gives guidelines to maximize network robustness by recognizing nodes that should not be in the interconnection between the two networks. The work of Yagan et al. [13] focuses on cyber-physical interdependent systems, and shows that a uniform allocation of bidirectional edges to all nodes in the system is the optimal design strategy against random attacks on unknown topologies.

In all the aforementioned works, the authors assume that propagation from one node to another follows an epidemic model, where an affected node can propagate the phenomena to any of its neighbors, regardless of the status of its other neighbor nodes. By contrast, there is another line of research, exemplified by the works of Kempe et al. [10], and Watts at al. [20], which considers threshold-based propagation models, according to which a node can be affected by the propagating phenomenon only if the number of its affected neighbors exceeds a given threshold. The works of Lee et al. [14], [15] extend the threshold model to the case of multiple layers of interdependent networks, in particular in [14] with heterogeneous response of nodes from one layer to another.

Our work tackles important aspects that were not addressed in previous work in a single unified framework. First, we generalize the role of interconnectivity links, which may reflect either a peer role or a functional dependency relationship between nodes. Second, we study the temporal process of propagation, considering a model which includes an aleatory delay in the propagation which has a significant impact, in terms of speed and extent of propagation with time. Third, we address the study of propagation from a network engineering perspective, highlighting the impact on propagation of some structural properties of nodes and their interconnections. To provide better understanding of phenomena propagation in interdependent networks, we propose a general model that can cover a wide range of phenomena, from failure propagation to information diffusion, and can be applied to any interdependent network system, regardless of network topology, type of phenomena, and inter-connectivity model. As also pointed out in the work by Kitsak et al. [21], classic metrics of centrality do not always identify the most influential spreaders in a complex network. We introduce a new centrality metric that successfully identifies spreaders even in our general model of propagation.

We clarify that, despite the generality of the proposed framework, our work cannot be applied to model interdependent networks for which failures do not propagate according to a topological model, such as the power grid. In fact, although most of the literature in this area in the last decade have assumed an epidemic topological model (neighbor to neighbor propagation) even for the propagation of failures along the nodes of a power grid, more recent work by Hines et al. [22] highlighted that in this type of network the propagation model is very specific. In fact, the propagation of failures proceeds along the network, spreading between points that may be far apart from each other and not directly connected.

3 Interconnected Network Model

Table 1 summarizes the notation used in the paper. Let us consider two interconnected networks, represented by two undirected graphs $G_X = (X, E_X)$ and $G_Y = (Y, E_Y)$, with n_X

TABLE 1
Nomenclature and Notation

Notation	Description
$G_{\mathtt{X}} = (X, E_{\mathtt{X}})$	graph of network X
$G_{\mathtt{Y}} = (Y, E_{\mathtt{Y}})$	graph of network Y
$n_{\mathtt{x}}$, $n_{\mathtt{y}}$	number of nodes in X and Y
$A_{\mathtt{XX}}$, $A_{\mathtt{YY}}$	adjacency matrixes of X and Y
$A_{\mathtt{XY}}$, $A_{\mathtt{YX}}$	matrixes of inter-links
$U_{\mathtt{X}}^{intra}(i) \subset X$	intra-neighbors of node $i \in X$
$U_{\mathtt{Y}}^{intra}(j) \subset Y$	intra-neighbors of node $j \in Y$
$U_{\mathtt{X}}^{inter}(i) \subset Y$	inter-parents of node $i \in X$
$U_{\mathtt{Y}}^{inter}(j) \subset X$	inter-parents of node $j \in Y$
$k_{\mathtt{xx}}(i) \in [0, 1]$	intra-propagation threshold to $i \in X$ (in % of affected intra-neighbors)
$k_{\mathtt{yy}}(j) \in [0, 1]$	intra-propagation threshold to $j \in Y$
$k_{\mathtt{xy}}(j) \in [0, 1]$	inter-propagation threshold to $j \in Y$
$k_{\mathtt{yx}}(i) \in [0, 1]$	inter-propagation threshold to $i \in X$
F_0	set of initial spreader nodes
$\gamma_{intra}(i)$	fraction of affected intra-neighbors of node i
$\gamma_{inter}(i)$	fraction of affected inter-parents of node i
$\gamma'_{intra}(i)$	number of affected intra-neighbors of node i
$\gamma'_{inter}(i)$	number of affected inter-parents of node i
$P_{prop}^{intra}(i)$	propagation probability from intra-neighbors to node i
$P_{prop}^{inter}(i)$	propagation probability from inter-parents to node i
$p_{\mathtt{max_X}}(i)$	propagation probability to $i \in X$ from intra-neighbors when all intra-neighbors are affected
$p_{\mathtt{max_Y}}(i)$	propagation probability to $j \in Y$ from intra-neighbors when all intra-neighbors are affected
$p_{\mathtt{max_{YX}}}(i)$	propagation prob. to $i \in X$ from inter-parents when they are all affected
$p_{\mathtt{max_{XY}}}(j)$	propagation prob. to $j \in Y$ from inter-parents when they are all affected
$K_{\mathtt{xx}}(i)$	intra-propagation threshold to $i \in X$ (in number of affected intra-neighbors)
$K_{\mathtt{yy}}(j)$	intra-propagation threshold to $j \in Y$
$K_{\mathtt{yx}}(i)$	inter-propagation threshold to $i \in X$
$K_{\mathtt{xy}}(j)$	inter-propagation threshold to $j \in Y$
S_T	state space of the propagation process
$s \in [0, 1]^{n_x+n_y}$	generic state of the propagation process
M	transition probability matrix of the process
$M\|_{s,s'}$	generic (s, s')-element of M
$I(condition)$	indicator function of *condition*
C_v	centrality of node v
$\alpha \in \{\alpha_s, \alpha_i, n_{\mathtt{x}}\}$	max # of nodes with inter-links from network X to Y, each with probability $P_{\mathtt{XY}}$
$\beta \in \{\beta_s, \beta_i, n_{\mathtt{y}}\}$	max # of nodes with inter-links from network Y to X, each with probability $P_{\mathtt{YX}}$

and $n_{\mathtt{Y}}$ nodes, respectively. We equivalently refer to these as networks X and Y. Without loss of generality, we assume that nodes belonging to the same network have peer roles. For this reason we use undirected links to represent connections of nodes of a same network and model them through a symmetric adjacency matrix. Notice that this assumption has no impact on the analytical model of propagation, which may as well work with directed links and asymmetric adjacency matrix. We denote with $A_{\mathtt{XX}} \in \{0,1\}^{n_{\mathtt{X}} \times n_{\mathtt{X}}}$ and $A_{\mathtt{YY}} \in \{0,1\}^{n_{\mathtt{Y}} \times n_{\mathtt{Y}}}$ the symmetric adjacency matrices representing the *intra-links* of networks X and Y, respectively. The two networks are interconnected by means of directed

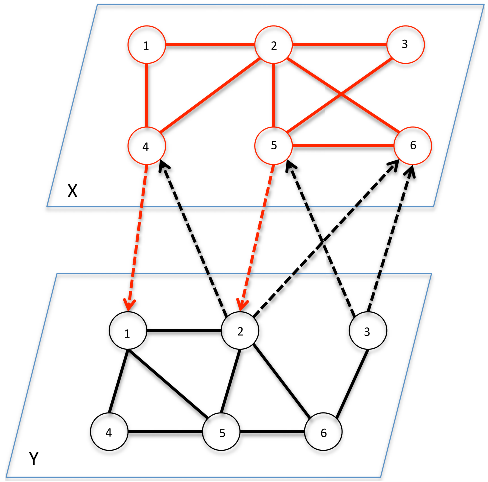

Fig. 1. Interdependent network model. Dashed-lines are intralinks, solid lines are interlinks.

inter-links, according to the inter-connection matrices $A_{\mathtt{XY}} \in \{0,1\}^{n_{\mathtt{X}} \times n_{\mathtt{Y}}}$ and $A_{\mathtt{YX}} \in \{0,1\}^{n_{\mathtt{Y}} \times n_{\mathtt{X}}}$. We use directed edges for inter-links to capture different models of interdependency between heterogeneous networks.

Two nodes connected through an intra-link are *intra-neighbors*. The set of *intra-neighbors* of node $i \in X$ is denoted by $U_{\mathtt{X}}^{intra}(i) = \{j \in X | (i,j) \in E_{\mathtt{X}}\}$, while the value of $|U_{\mathtt{X}}^{intra}(i)|$ is the *intra-degree* of node $i \in X$. Similar notations are introduced for network Y.

Given a node $i \in X$, we define the set of *inter-parents* of i, as $U_{\mathtt{X}}^{inter}(i) = \{j \in Y | A_{\mathtt{YX}}(j,i) = 1\}$. We refer to $|U_{\mathtt{X}}^{inter}(i)|$ as the *inter-degree* of node $i \in X$. Similar notation is used for the interconnecting edges from network X to network Y. Fig. 1 shows an example of two interconnected networks X and Y, of six nodes each. Dashed lines are directed inter-links, and solid lines are intra-links. As an example, the presence of the directed inter-link from node 4 of X to node 1 of Y denotes that node 4 is an inter-parent of node 1. Similarly, the intra-neighbors of node 5 of network X are nodes 2, 3, 6 of X, while its only inter-parent is node 3 of Y.

3.1 Enabling Conditions for Propagation to a Node

We use the neutral term *phenomenon* to refer to either desired or undesired propagating events, as a cascade of failures or the diffusion of a piece of information or an opinion. We also generically term as *affected* any node that has been reached by the propagating phenomenon.

In the following we introduce a threshold-based propagation model. This is a general parametric model that incorporates previous models as special cases. According to this model a node may become affected by the phenomenon only if the number of affected neighbors exceeds a given threshold, and with a probability that is proportional to this number. This reflects some real scenarios, such as the spread of an opinion or the adoption of a new technology/product in a social networks. In both cases the propagation is more likely to occur when the number of affected nodes in the network increases.

Previous works also contributed some threshold based propagation models [10], [11], [12], [14], [20], [23]. Most of these only considered a single network, with homogeneous

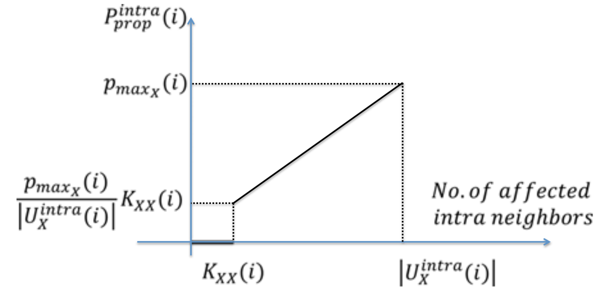

Fig. 2. Probability of phenomena propagation from intra-neighbors to node $i \in X$ within one time step.

propagation thresholds. Unlike these works, we consider interconnected networks, in which nodes have heterogeneous propagation thresholds and propagation probabilities across the two networks and within a single network. We address network heterogeneity by differentiating between the way the propagation occurs within a network, called *intra propagation*, and across two heterogeneous networks, referred to as *cross propagation*.

Our model is general for the following reasons:

1) It covers most of the traditional models of propagation, from contagion/epidemic models [24], [25] to the threshold cascade model proposed by Kleinberg et al. [12].
2) It does not assume a specific network topology, nor any kind of inter-dependency in contrast to the previous works which usually assume a specific scenario.
3) It considers a flexible probabilistic model of propagation that can be used for any type of network with heterogeneous nodes by defining different values for the thresholds and propagation probabilities.
4) Unlike some previous work by Havlin et al. [2], our model does not rely on any assumption of having only one type of propagation in each time step. Notice that this assumption has no impact on the percolation properties (percolation threshold and size of the giant component). Nevertheless, to properly study the dynamics of the propagation process, it is essential to consider that the phenomena can propagate simultaneously in any direction, inside one network and from one network to the other, which reflects real-world scenarios.

Similar to the work on bootstrap percolation [17], [18], we assume that initially only a subset F_0 of nodes is affected by the phenomenon. Unlike these works, we study the time process of propagation among the nodes of the two networks. We consider discrete time steps. The propagation of a phenomenon from one node to the others may occur in one step or be delayed and occur after multiple time steps.

For the two interdependent networks, we introduce two different threshold functions $k_{\mathtt{xx}}(i) \in (0,1]$ for $i \in X$ and $k_{\mathtt{yy}}(j) \in (0,1]$ for $j \in Y$, to model the propagation across nodes of the same network. We also introduce two other thresholds $k_{\mathtt{yx}}(i) \in (0,1]$ for $i \in X$, and $k_{\mathtt{xy}}(j) \in (0,1]$ for $j \in Y$, to model the propagation across the two interdependent networks, from network X to network Y and vice versa, respectively.

We denote by $P_{prop}^{intra}(i)$ the probability of propagation to node i, due to the same network neighbors, i.e., intra-neighbors, and by $P_{prop}^{inter}(i)$ the propagation probability related to the inter-parents. In particular, assuming that a fraction $\gamma_{intra}(i)$ of intra-neighbors of node $i \in X$ are already affected by the phenomenon, the probability that the phenomenon propagates to $i \in X$ within one time step is $P_{prop}^{intra}(i)$, and is calculated as follows:

$$P_{prop}^{intra}(i) = \begin{cases} \frac{\gamma'_{intra}(i)}{|U_{\mathtt{X}}^{intra}(i)|} \cdot p_{\mathtt{max_X}}(i) & \text{if } \gamma'_{intra}(i) \geq K_{\mathtt{xx}}(i) \\ 0 & \text{if } \gamma'_{intra}(i) < K_{\mathtt{xx}}(i), \end{cases} \tag{1}$$

where $K_{\mathtt{xx}}(i) = \lceil k_{\mathtt{xx}}(i) \cdot |U_{\mathtt{X}}^{intra}(i)| \rceil$ and $\gamma'_{intra}(i) = \lceil \gamma_{intra}(i) \cdot |U_{\mathtt{X}}^{intra}(i)| \rceil$ is the number of affected intra-neighbors of node i. Here $p_{\mathtt{max_X}}(i)$ represents the probability that node $i \in X$ is affected by the phenomenon within one time step, when all of its intra-neighbors are already affected.

Fig. 2 shows the probability that a phenomenon propagates to node $i \in X$ within one time step, as a function of the number of intra-neighbors that are already affected. We see that if the number of affected intra-neighbors of node i is less than the threshold, the probability of propagation to node i is zero. By contrast, when the number of involved intra-neighbors exceeds the threshold, the probability of propagation increases linearly with the number of affected intra-neighbors.

Similarly, node $i \in X$ may be affected by the propagating phenomenon due to the fact that some inter-parents are also affected. Therefore, denoting with $\gamma'_{inter}(i)$ the number of affected inter-parents of node i, we define:

$$P_{prop}^{inter}(i) = \begin{cases} \frac{\gamma'_{inter}}{|U_{\mathtt{X}}^{inter}(i)|} \cdot p_{\mathtt{max_{yx}}}(i) & \text{if } \gamma'_{inter}(i) \geq K_{\mathtt{yx}}(i) \\ 0 & \text{if } \gamma'_{inter}(i) < K_{\mathtt{yx}}(i), \end{cases} \tag{2}$$

where $K_{\mathtt{yx}}(i) = \lceil k_{\mathtt{yx}}(i) \times |U_{\mathtt{X}}^{inter}(i)| \rceil$ and $\gamma'_{inter}(i) = \lceil \gamma_{inter}(i) \times |U_X^{inter}(i)| \rceil$, and where $\gamma_{inter}(i)$ is the fraction of currently affected inter-parents of node i. The parameter $p_{\mathtt{max_{YX}}}(i)$ represents the probability that node $i \in X$ is involved in the propagating phenomenon within one time step after all inter-parents of i inside network Y have been affected. We use a similar notation for node $j \in Y$ by defining equations analogous to Equations (1) and (2), where we use the thresholds $k_{\mathtt{yy}}(j)$ and $k_{\mathtt{xy}}(j)$ to express the required fraction of intra-neighbors and inter-parents of node i that must be affected before i can become affected with positive probability. For the sake of brevity, we will use $p_{\mathtt{max}}(\cdot)$ to refer to the case where the transition probabilities are homogeneously defined, namely $p_{\mathtt{max_X}}(\cdot) = p_{\mathtt{max_Y}}(\cdot) = p_{\mathtt{max_{XY}}}(\cdot) = p_{\mathtt{max_{YX}}}(\cdot)$.

Our propagation model includes the characteristics of previous models in a unique general framework, as we highlight in the following examples.

The random threshold model introduced in [10], [11], can be obtained by modeling a single network X and setting $k_{\mathtt{xx}}(i)$ to a random value, for each $i \in X$.

The viral spread (one-to-one) epidemic model common to many works [1], [2], [3], [4], [6] can be obtained by setting

$k_{\mathtt{xx}}(i) = \frac{1}{|U_{\mathtt{X}}^{intra}(i)|}$, $k_{\mathtt{yx}}(i) = \frac{1}{|U_{\mathtt{X}}^{inter}(i)|}$ for any node $i \in X$, and $k_{\mathtt{yy}}(j) = \frac{1}{|U_{\mathtt{Y}}^{intra}(j)|}$, and $k_{\mathtt{xy}}(j) = \frac{1}{|U_{\mathtt{Y}}^{inter}(j)|}$ for $j \in Y$.

The partial dependency model presented in [2], [5] (where only a fraction of nodes in X has support links from network Y and vice versa), multiple-support dependency relation model presented in [6], and same-degree mutual dependency model presented in [4] (where a one-to-one dependency is considered between nodes with identical degree in the two networks) can be obtained in a straightforward manner by appropriately modeling the adjacency matrices, and setting $p_{\mathtt{max_{XY}}}(\cdot) = p_{\mathtt{max_{YX}}}(\cdot) = 1$ and $k_{xy}(\cdot) = k_{yx}(\cdot) = 1$.

4 The Propagation Process

We model the temporal evolution of the phenomenon propagation with a Markov model. We denote by S_T the set of states of the model, where each state is defined as a vector $s = (\overbrace{s_1 s_2 \dots s_{n_{\mathtt{x}}}}^{\in X} \overbrace{s_{n_x+1} \dots s_{n_x+n_y}}^{\in Y})$, in which s_k (for $k \leq n_x$) is 1 if node $k \in X$ is affected, and 0 if it is not affected by the propagating phenomenon. Similarly, for $k > n_{\mathtt{x}}$, s_k is 1 if node $k - n_{\mathtt{x}}$ of network Y is affected and 0 otherwise. Therefore, the initial state of the propagation process is $s_{\mathtt{initial}} = (s_1 s_2 \dots s_{n_x} s_{n_{\mathtt{x}}+1} \dots s_{n_{\mathtt{x}}+n_{\mathtt{y}}})$, where $s_k = 1$ if $k \leq n_x \wedge > k \in X \cap F_0$ or $k > n_{\mathtt{x}} \wedge (k - n_{\mathtt{x}}) \in Y \cap F_0$, while $s_k = 0$ otherwise.

According to our propagation model, a node can be affected by the phenomenon only if the number of its affected intra-neighbors and/or inter-parents exceeds a given threshold. Therefore, not all the binary vectors of $n_{\mathtt{x}} + n_{\mathtt{y}}$ elements represent a feasible state of the process. Based on the previous definitions, it is straightforward to calculate the one-step transition probability matrix of the process $M \in \{0,1\}^{|S_T| \times |S_T|}$, whose element $M|_{s_i,s_j}$ gives the probability that the network transits from state s_i to state s_j.

4.1 Transition Probabilities of the Propagation Process

In the following we describe how to calculate the one-step transition probabilities of the propagation process.

We recall that M denotes the transition probability matrix, and $M|_{s,s'}$ its generic element, corresponding to the transition probability from state s to state s'. We also let $\Delta s = s' - s$. We will refer to the jth elements of vectors Δs and s by Δs_j and s_j, respectively. Notice that, in order to calculate $M|_{s,s'}$ we are only interested in transitions for which $\Delta s_j = 0, 1, \ \forall j = 1, \dots, n_{\mathtt{x}} + n_{\mathtt{y}}$.

We denote by $I(condition)$ an indicator function, which is equal to 1 if the value of the boolean *condition* is `true`, and 0 otherwise. The generic element of M can be calculated as follows:

$$M|_{s,s'} = \prod_{j=1}^{n_x+n_y} \Big[f_j \cdot I(\Delta s_j = 0) + f'_j I(\Delta s_j = 1) \Big], \tag{3}$$

where f_j denotes the probability that there is no change in the jth component of the state when moving from state s to state s', and f'_j is the probability that there is such a change. Notice that if there is no change in a state component, it means that the related node is either already affected, or it is unaffected and remains so. The latter case, may be due to the fact that the conditions for propagation described in Section 3.1 are not valid, or to the fact that the propagation, although enabled by the propagation conditions, did not occur in the current time step.

We now calculate the value of f_j and f'_j by separating the terms related to network X and Y. In particular, f_j, the probability of having a change in the jth component when the status change from s to s', is as follows:

$$f_j = \begin{cases} g_{\mathtt{x}}(j) & \text{if } j \leq n_{\mathtt{x}} \wedge s_j = 0, \\ g_{\mathtt{y}}(j) & \text{if } j > n_{\mathtt{x}} \wedge s_j = 0, \\ 1 & \text{if } s_j = 1. \end{cases}$$

The term $g_{\mathtt{x}}(j)$ can be calculated as

$$\begin{aligned} g_{\mathtt{x}}(j) &= I(\gamma_{intra}(j) < k_{\mathtt{xx}}(j)) \cdot I(\gamma_{inter}(j) < k_{\mathtt{yx}}(j)) \\ &+ I(\gamma_{intra}(j) \geq k_{\mathtt{xx}}(j)) \cdot I(\gamma_{inter}(j) < k_{\mathtt{yx}}(j)) \cdot \bar{P}_{prop}^{intra}(j) \\ &+ I(\gamma_{intra}(j) < k_{\mathtt{xx}}(j)) \cdot I(\gamma_{inter}(j) \geq k_{\mathtt{yx}}(j)) \cdot \bar{P}_{prop}^{inter}(j) \\ &+ I(\gamma_{intra}(j) \geq k_{\mathtt{xx}}(j)) \cdot I(\gamma_{inter}(j) \geq k_{\mathtt{yx}}(j)) \cdot \\ &\cdot \bar{P}_{prop}^{intra}(j) \bar{P}_{prop}^{inter}(j), \end{aligned} \tag{4}$$

where $\bar{P}_{prop}^{intra}(j) = 1 - P_{prop}^{intra}(j)$ and $\bar{P}_{prop}^{inter}(j) = 1 - P_{prop}^{inter}(j)$. We recall that $P_{prop}^{intra}(j)$ and $P_{prop}^{intra}(j)$ are defined in Equations (1) and (2), respectively.

In order to calculate the term $g_{\mathtt{y}}(j)$, we define $j' = j - n_x$ and apply a formula analogous to the one in Equation (4).

The term f'_j denotes the probability that there is a change in the jth component of the state vector, when going from state s to state s'. As we did for the term f_j we split f'_j in the contributions related to the two networks. Therefore,

$$f'_j = \begin{cases} g'_{\mathtt{x}}(j) & \text{if } j \leq n_{\mathtt{x}} \wedge s_j = 0, \\ g'_{\mathtt{y}}(j) & \text{if } j > n_{\mathtt{x}} \wedge s_j = 0, \\ 0 & \text{if } s_j = 1. \end{cases}$$

Hence the term $g'_x(j)$ can be calculated as follows:

$$\begin{aligned} g'_{\mathtt{x}}(j) &= I(\gamma_{intra}(j) \geq k_{\mathtt{xx}}(j)) \cdot I(\gamma_{inter}(j) \geq k_{\mathtt{yx}}(j)) \cdot \\ &\cdot \big[P_{prop}^{intra}(j) \cdot P_{prop}^{inter}(j) + \bar{P}_{prop}^{intra}(j) \cdot P_{prop}^{inter}(j) \\ &+ P_{prop}^{intra}(j) \cdot \bar{P}_{prop}^{inter}(j) \big] \\ &+ I(\gamma_{intra}(j) < k_{\mathtt{xx}}(j)) \cdot I(\gamma_{inter}(j) \geq k_{\mathtt{yx}}(j)) \cdot P_{prop}^{inter}(j) \\ &+ I(\gamma_{intra}(j) \geq k_{\mathtt{xx}}(j)) \cdot I(\gamma_{inter}(j) < k_{\mathtt{yx}}(j)) \cdot P_{prop}^{intra}(j). \end{aligned} \tag{5}$$

Finally, by considering $j' = j - n_{\mathtt{x}}$, we can write, for the nodes of network Y an Equation analogous to Equation (5).

Notice that, for a fixed set of network parameters, if the intra-propagation threshold (k_{yy}, k_{xx}) and/or inter-propagation threshold (k_{xy}, k_{yx}) are smaller, it is more likely that the propagation conditions of Section 3.1 are satisfied. This corresponds to an increase of f'_j with respect to f_j. Indeed, from Equation (3) we see that to increase the speed of propagation, we need to have $f'_j >> f_j$ to let the second term of Equation (3) dominate the first. Also by fixing all network parameters and increasing the value of $p_{\mathtt{max_{**}}}(\cdot)$, we can predict that $f'_j > f_j$, since f'_j is an increasing function of $P^{intra}_{prop}, P^{inter}_{prop}$ while f_j is an increasing function of $\bar{P}^{intra}_{prop}, \bar{P}^{inter}_{prop}$ and therefore, a decreasing function of $P^{intra}_{prop}, P^{inter}_{prop}$. In Section 7 we provide extensive simulations to validate these claims.

4.2 Expected Time to Absorption

The phenomena propagation can be seen as an absorbing Markov process, in which the absorbing states are those in which either all nodes are affected, or no unaffected node meets the propagation condition described in Section 3.1.

Notice that, depending on the value of the thresholds k_{xx}, k_{yy}, k_{xy} and k_{yx}, and the set of initial starters F_0, we may have different absorbing states.

Standard techniques for the analysis of Markov processes [26] can be applied to calculate the expected time to absorption of the proposed Markov model.

Let t be the number of non-absorbing state. Let $Q \in [0,1]^{t\times t}$ be a square matrix giving the transition probabilities from non-absorbing to non-absorbing states. We define the *fundamental matrix* $N \triangleq (I-Q)^{-1} = I + Q + Q^2 + \cdots$, where $\lim_{k\to\infty} Q^k = 0$.

Let $\hat{\mathbf{t}} \in \mathbb{R}^t$ be a column vector representing the mean time to absorption, where $\hat{t}_i$ is the mean time to absorption when the initial state is the ith non absorbing state. The value of $\hat{t}_i$ is the sum of the elements of the ith row of N. Therefore $\hat{t}_i = \sum_{j=1}^{t} N_{ij}$.

5 Discussion on Network Design

We recall that the elements of the transition probability matrix introduced in Section 4.1 depend on the adjacency matrices of the two interdependent networks, on the thresholds k_{xx}, k_{yy}, k_{yx}, k_{xy}, and on the parameters $p_{\mathtt{max_X}}(\cdot)$, $p_{\mathtt{max_Y}}(\cdot)$, $p_{\mathtt{max_{YX}}}(\cdot)$, and $p_{\mathtt{max_{XY}}}(\cdot)$. We also recall that all of these parameters may have different values for each node.

All the above mentioned parameters can be tuned in the network design phase to create a network with the desired characteristics in terms of propensity to propagate phenomena or robustness to them.

For example, in the context of failure propagation, higher threshold values correspond to nodes that are less prone to be affected by the propagation from either intra-neighbor or inter-parent nodes. In terms of network design, to have a node with a higher threshold we may adopt systems where nodes are dependent on each other with some level of redundancy.

In contrast, in the context of a social network, a higher threshold in the propagation model may reflect a situation in which individuals are not easily influenced and corroboration of consensus is needed before propagation can occur.

Similarly, the parameters $p_{\mathtt{max_X}}(\cdot)$, $p_{\mathtt{max_{YX}}}(\cdot)$, or $p_{\mathtt{max_Y}}(\cdot)$ and $p_{\mathtt{max_{XY}}}(\cdot)$, represent the capability of a node to foster or prevent the spread of a phenomenon. For instance, in the propagation of failure, having a node with a high value of these parameters implies that this node is more vulnerable and tends to be affected by the failure with a higher probability. As a design criterion, if some nodes play a critical role in the connectivity of the network, it will be a good design strategy to invest more effort in making these nodes more robust and reliable, which lowers the values of the above mentioned propagation probability parameters and implies a slower, or less likely propagation.

In the context of information spreading in a social network, a node with high values of these propagation probabilities may be useful as it is rapidly affected, as soon as the propagation conditions are satisfied.

This analysis gives some hints on how to design networks that may show a better behavior in terms of propagation, in both cases in which the propagation is either a desired or an undesired event.

In order to control the propagation speed of the network, a design strategy may consist of designing the network elements and their inter-connectivity so as to have the desired values of the propagation parameters. Nevertheless, this type of design may be costly, and it may not even be necessary at every node. In a successful and cost efficient design strategy we are interested in determining a subset of nodes on which to intervene by adopting the necessary measures to modify their propagation parameters with respect to the rest of the network.

Hence it is of particular importance to be able to determine which nodes play a critical role in the propagation and why.

In Section 7 we discuss experiments that determine the impact of the node degree in affecting the propagation, with particular focus on the nodes that are the endpoints of interconnecting edges between the interdependent networks. To this purpose we investigate the impact of different interconnectivity patterns based on the degree of the interconnecting nodes. In particular, we experimentally investigate the effect of interconnecting the networks by means of the nodes with minimum or maximum degree, showing how the degree of the interconnecting nodes plays a major role in determining the speed of propagation across the interdependent networks.

Nevertheless, it must be noted that the degree of a node is just one of several possible metrics of node *centrality* that can be adopted to determine the propensity of a node to propagating phenomena.

In the next section we discuss several classic metrics of centrality and we introduce a new metric specifically tailored to propagation models based on threshold conditions. In our experiments in Section 7 we compare the efficacy of the centrality metrics in determining the most influential nodes. We perform these comparisons by altering the propagation parameters of the nodes with high centrality, according to different metrics. We thus motivate the adoption of efficient design strategies to create networks with the desired propagation characteristics, by tackling the specific characteristics of only the nodes with high centrality.

5.1 Proposed Centrality Metric

In graph theory and network analysis, the notion of "centrality indicator" is used to refer to metrics that identify the most important nodes within a graph.

Although classic centrality metrics [27], [28], [29] are helpful in capturing the importance of the nodes inside a single network, they may fail to quantify the importance of the nodes in the interconnection of two heterogeneous networks. Moreover, classic metrics do not consider the role of a node neighborhood in affecting the spread across the network according to the propagation conditions described in Section 3.1. For example, a node may be central according to classic centrality notions but it may still be unable to spread the information across the network due to an insufficient number of nodes satisfying the propagation conditions along these paths.

Considering the two interconnected networks as a single network only partially solves the limited capability of classic metrics to capture the relevance of specific nodes to the propagation. In contrast, considering the two networks as a whole implies neglecting the differences between propagation inside one network and cross propagation across the two networks. For example, in many real-world applications, after a phenomena happens in one of the two networks, the objective is to stop the propagation to the other network. Therefore, in these cases, we only need to determine the important nodes for the propagation of the phenomena from one network to the other one.

To determine the important nodes for the cross propagation, we investigated their evolution over time. We noticed that whenever a phenomenon passes the border of one network and reaches the borderline nodes of the other, the number of involved nodes increases significantly (see for example Fig. 9). This confirms that the nodes that interconnect the two networks can have a larger impact on the phenomena propagation and therefore are more important. In fact, the borderline nodes act as a firewall. Based on these observations, we propose a new centrality metric, called path-degree (PD) centrality, that reflects the importance of the node in its local network and in the context of interdependent networks, and at the same time considers our threshold model of propagation.

Unlike the traditional centrality metrics where the focus is on the impact of a node on propagation inside one single network, PD centrality focuses on propagation from one network to the other network. Therefore, the proposed metric gives a higher importance to the nodes that are located on the border/close to the border of the two networks, thus, are acting as hubs for the paths between the two networks. Inspired by the definition of betweenness centrality [27], path-degree centrality for a node v is defined as

$$C_v = \begin{cases} \sum_{s\in X, t\in B_Y s\neq t\neq v} & \delta_{st}(v) \quad if\ v\in X, \\ \sum_{s\in Y, t\in B_X s\neq t\neq v} & \delta_{st}(v) \quad if\ v\in Y, \end{cases} \tag{6}$$

where $\delta_{st}(v)$ represents the total number of shortest paths from a node s to node t, that pass through node v. B_X and B_Y are the set of borderline nodes, in X and Y, respectively.

Unlike the classic notion of betweenness centrality, we adopt a weighted measure of link length. The length $l(p)$ of a path p is calculated as follows:

$$l(p) = \sum_{\forall n_i \in p\setminus\{s\}} \lceil deg\,(n_i) \times k\,(n_i)\rceil, \tag{7}$$

where

$$deg(n_i) = \begin{cases} |U^{intra}(n_i)| & \text{if} \quad n_i \in A \wedge n_{i-1} \in A \\ |U^{inter}(n_i)| & \text{if} \quad n_i \in A \wedge n_{i-1} \in B \end{cases}$$

$$k(n_i) = \begin{cases} k_{\mathsf{aa}} & \text{if} \quad n_i \in A \wedge n_{i-1} \in A \\ k_{\mathsf{ab}} & \text{if} \quad n_i \in A \wedge n_{i-1} \in B, \end{cases}$$

where A, B can represent any of the two networks X, Y, and $A \neq B$. Similarly, k_{aa} may represent k_{xx} or k_{yy}.

The use of node degree as a weight in the calculation of the path length, is motivated by the propagation model introduced in Section 3.1. According to this model, the impact that a single node can have on one of its neighbors depends on the degree of the neighbor. In fact, if a node is the only intra-neighbor of another node, and it is already affected, there is a high chance that the intra-neighbor node becomes affected too, according to the propagation conditions of Section 3.1. In contrast, a node with multiple intra-neighbors is less likely to be affected by a single neighbor, because other intra-neighbors need to be affected to validate the propagation conditions.

Based on our metric of path length expressed by Equation (7), increasing the degree of the nodes located on the path (for a fixed value of $k(n_i)$), increases the length of the path, and decreases the centrality of the node.

6 Details on the Simulation Environment

Since the state space of the Markov model increases exponentially with the number of nodes in the two networks, we use our Markov model for evaluating medium size networks only. For larger size networks, we resort to a network simulator based on MATLAB [30].

We will use this simulator to evaluate the impact of network type, coupling models, set of initial spreaders, and other aspects on the speed of propagation. Let F_0 be the set of initial spreaders, which is the set of nodes that are affected by the phenomenon from the beginning. We want to evaluate the number of nodes that are affected over time.

Unlike previous works in this area, [3], [4], [19], [31], that study asymptotic statistical properties such as size of the remaining giant connected component after a failure, we are not looking at the asymptotic behavior of the phenomenon propagation. Instead we investigate the propagation of the phenomenon over time, and which nodes are most likely to be affected. The goal is to identify influential spreaders and to determine useful inter-connection methods that can help us to design more efficient interdependent networks.

6.1 Network Type

In the experiments, we investigate propagation in three well-known artificial network models, namely Scale-Free (SF) [32] which reflects the characteristics of preferential attachment in WWW links, Small-World (SW) [33] indicating some types of social networks, and Erdos-Renyi (ER) [34] used for completeness in the analysis of random graphs. The study of these networks gives us a broad insight on how phenomena spreads across the nodes of different topologies and allows us to perform extensive simulations under a wide range of experimental scenarios, with varying structural parameters of the network model. In addition, we extend our study to real network topologies to highlight the effect of variations of propagation parameters. To this end, we consider real network topologies taken from from the Center for Applied Internet Data Analysis (CAIDA) resource collection [35].

In each experiment where we use artificial networks, the graphs G_{X} and G_{Y} are generated according to one of these three network growth models.

Scale-Free. In a the scale-free network the degree distribution is defined by a power law, so most nodes have relatively few links but a few nodes (called hubs) have a high number of links. The contribution of the hubs to the overall connectivity is very high with respect to those of nodes with lower degree. This model is often used to represent the WWW in which most web pages have only a few links connecting to them, but sites like Google and Yahoo have a very large number of hyperlinks pointing to them [32]. To generate a Scale-Free network, a seed network of few nodes, n_{seed}, is generated initially and additional nodes are added with a preferential attachment procedure. The number of links a new node can make to the existing network nodes, m_{links}, can control the average degree of the network.

Small-World. In this type of network, most nodes can be reached from every other node via short paths. The idea behind generating graphs with small-world characteristics is to start with order and randomize a bit. Many empirical graphs show small-world characteristics, e.g., Social networks, wikis such as Wikipedia, and gene networks [33]. In our simulations, we start with a ring of n vertices in which each vertex is connected to its k_{sw} nearest neighbors, for a given k_{sw}. Then, each edge is rewired with a given probability p_{rw} by choosing randomly a new vertex to connect.

Erdos-Renyi. In this model, a graph is constructed by considering a set of nodes and connecting them randomly. An edge is added to each pair of nodes with probability p, therefore, the distribution of the degree of any particular vertex is binomial. An Erdos-Renyi network has the property that the majority of nodes have a degree that is close to the average degree of the overall network and that the deviation from the average is rather limited. The distribution of the links follows a Poisson distribution [34].

Where necessary, we will tune the network parameters so as to obtain networks with a desired value of the average node degree. An ER network with average degree d and n nodes, can be generated by setting the probability of adding an edge to $p = \frac{d}{n}$. In the case of the SF network, we obtain a network with the desired average degree by setting $n_{seed} = 5$ and varying the number of links, m_{links}, to which each new node is attached. To generate a Small-World network with a given average degree, we fixed the rewiring probability to $p_{rw} = 0.4$ starting from an initial ring-shaped network with k_{sw} neighbors for each node.

6.2 Inter-Connectivity (Coupling) Models

To cover a wide range of inter-connectivity models, we use two schemes: a) randomized model, and b) designed model. To evaluate the impact of the level of coupling between the two networks $G_{\mathtt{X}}$ and $G_{\mathtt{Y}}$, for each set of models, we define three different levels of inter-connectivity, namely high inter-connectivity, sparse inter-connectivity, and intermediate inter-connectivity. For simplicity, in the remainder of the paper, we will call *borderline nodes* all the nodes that are endpoints of inter-connecting links between the two networks. In the following, we explain each model in detail.

Randomized Model. Borderline nodes are selected randomly. We recall that interconnecting links are directed edges. In this model, some nodes of network $G_{\mathtt{X}}$ are selected randomly. Let $\alpha \in [1, n_{\mathtt{x}}]$ be the number of such nodes. The selected nodes have an outgoing link to each node of network $G_{\mathtt{Y}}$ with probability $P_{\mathtt{XY}}$. Similarly, β nodes of network $G_{\mathtt{Y}}$ are selected randomly, and each selected node can have an outgoing link to each node of network $G_{\mathtt{X}}$ with probability $P_{\mathtt{YX}}$.

In our experiments, we distinguish three degrees of coupling, by setting the values of $\alpha \in \{\alpha_s, \alpha_i, n_{\mathtt{x}}\}$ and $\beta \in \{\beta_s, \beta_i, n_{\mathtt{y}}\}$ accordingly, with $\alpha_s < \alpha_i < n_{\mathtt{x}}$ and $\beta_s < \beta_i < n_{\mathtt{y}}$.

1) High inter-connectivity: $\alpha = n_{\mathtt{x}}$ and $\beta = n_{\mathtt{y}}$.
2) Intermediate inter-connectivity: $\alpha = \alpha_i$ and $\beta = \beta_i$.
3) Sparse inter-connectivity: $\alpha = \alpha_s$ and $\beta = \beta_s$.

Designed Model. In this model, the borderline nodes are selected on the basis of some metrics. We believe that a metric of centrality is a significant choice to engineering the network so as to foster or reduce the spread of a phenomenon between the two interconnected networks. For example, let us consider the case of two interconnected social networks $G_{\mathtt{X}}$ and $G_{\mathtt{Y}}$. By interconnecting nodes with higher centrality in the two networks we can speed up the flow of information. For the results shown in this paper, we determine the set of borderline nodes on the basis of the intra-degree of nodes. Depending on the intra-degree of the borderline nodes, we define three categories of interest:

Min-Min Model. α nodes of network X with minimum degree are connected to β nodes of network Y with minimum degree.

Min-Max Model. α nodes of network X with minimum degree are connected to β nodes of network Y with maximum degree.

Max-Max Model. α nodes of network X with maximum degree are connected to β nodes of network Y with maximum degree.

6.3 Initial Spreaders

The choice of the nodes that are initially involved in the phenomena may have a considerable impact on the speed of propagation. To characterize this impact, we investigate two different ways to define the set of initial spreaders F_0. In the proposed Markov model the state of these nodes is set to one at time $t = 0$.

Randomized Set. In this model, the initial spreaders are randomly selected.

Designed Set. In this model, the initial spreaders are the nodes with either maximum or minimum degree. This setting could reflect a real case scenario in which the spread of a failure may be the result of a targeted attack, or in the case of a social network, a situation in which the information is initially pushed to the most influential nodes with the purpose to have a fast diffusion.

6.4 Verification of the Simulation Model

In the following we initially verify the consistency of the simulator and the analytic model. As the analytic model does not scale with the size of the network we can only verify the simulation model comparing the results obtained with the theoretical Markov model and the simulator for small size interdependent networks.

We consider two interdependent networks, each including five nodes, i.e., $n_{\mathtt{x}} = n_{\mathtt{y}} = 5$. There are five outgoing links from the nodes of network X to the nodes of network

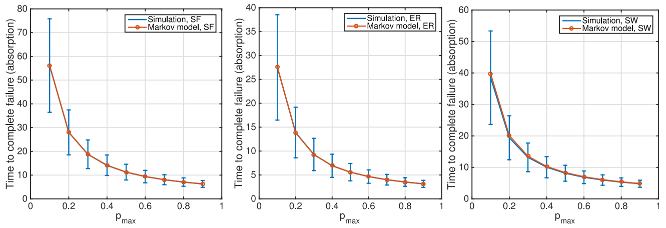

Fig. 3. Comparison between simulations and analysis (Markov model) for the three network types.

Y, and five outgoing links from the nodes of network Y to the nodes of X, that make the inter-connectivity between two networks. We set uniform propagation thresholds throughout the nodes of the same network: $k_{\mathtt{xx}} = k_{\mathtt{yy}} = 0.3$, $k_{\mathtt{xy}} = k_{\mathtt{yx}} = 0.4$. A node of network X is considered as the initial spreader. It is also assumed that $p_{\mathtt{max_X}} = p_{\mathtt{max_Y}} = p_{\mathtt{max_{XY}}} = p_{\mathtt{max_{YX}}} = 0.8$.

We assume that the set of initial spreaders includes one single node that is selected randomly from network X. We calculate the expected time to absorption using both the analytical model of Section 4 and the simulator.

Fig. 3 shows the expected time to complete propagation, which is the time until all the nodes of the two networks are affected by the phenomenon. In the Markov model this metric is calculated as described in Section 4.2. The expected time to absorption using our simulator is obtained by averaging over 10,000 runs. The error bars show the standard deviation of the experiments.

The results of this comparison are related to networks generated according to the three network models described in Section 6.1. In these experiments, the average degree of the nodes for SF, ER, SW are $1.6, 2.8, 3.2$, respectively. We see that for all the three types of network model, the simulation results are very close to the results obtained by Markov model. In Section 7 we perform extensive simulations on larger networks to evaluate the propagation process described in Section 4 and the design criteria discussed in Section 5.

7 Simulation Results

In the following, we analyze the propagation process under different network setups. We consider two interdependent networks $G_{\mathtt{X}}$ and $G_{\mathtt{Y}}$, with 100 nodes each, $n_{\mathtt{x}} = n_{\mathtt{y}} = 100$. Where not otherwise stated, we configure the propagation conditions defined in Section 3.1 according to the following threshold values, uniformly set for all the nodes inside each network: $k_{\mathtt{xx}} = k_{\mathtt{yy}} = 0.3, k_{\mathtt{xy}} = k_{\mathtt{yx}} = 0.4$. We investigate the percentage of the nodes that are affected by the propagating phenomenon over time. For all results provided in this section, a horizon of $T = 200$ time steps is considered. Unless explicitly mentioned otherwise, we generate networks X and Y with average degrees of 4 or 6, by tuning the network growth parameters as described in Section 6.1. Our results represent the average of 500 random topologies. We assume that the two networks are generated according to the same network model, either ER, SF or SW.

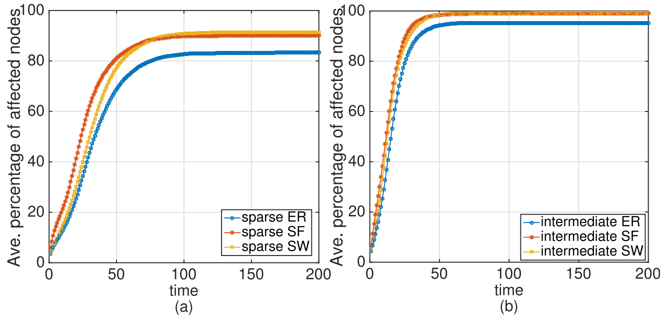

Fig. 5. Different network models: Propagation process over time, Average degree of nodes = 4, F_0 = 4 nodes of network X with max degree.

7.1 On the Impact of the Network Model and of the Choice of the Initial Spreaders

In this experiment, we investigate the effect of network model on the speed of propagation. To have a fair comparison between different network models, we consider networks with same node average degree. In all the following experiments we set $p_{\mathtt{max}}(\cdot) = 0.8$. We consider different levels of random inter-connectivity, for which we set $P_{\mathtt{XY}} = 0.05, \alpha_s = \beta_s = 2, \alpha_i = \beta_i = 8$ for the inter-connectivity models. The results are shown for three different network models, randomized and designed initial spreaders set, and different levels of coupling between the two networks.

In all the experiments of this section, shown in Figs. 4, 5, 6, and 7, we analyze the propagation of a phenomenon in the three types of network models and show the percentage of affected nodes over time. In Figs. 4 and 5 we consider networks with an average node degree of 4, and four initial spreaders, with sparse and intermediate inter-connectivity, respectively. In the scenario of Fig. 4, where the initial spreaders are selected randomly, the SW network generally produces a faster propagation with respect to the other two network models. The ER model is the slowest. In contrast, in Fig. 5 we show that when we select the initial spreaders as the four

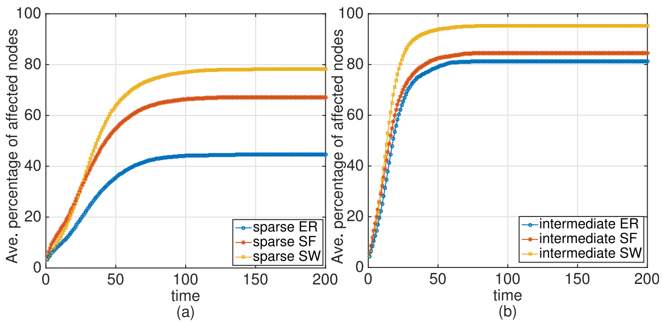

Fig. 4. Different network models: Propagation process over time, Average degree of nodes = 4, F_0 = 4 random nodes of network X.

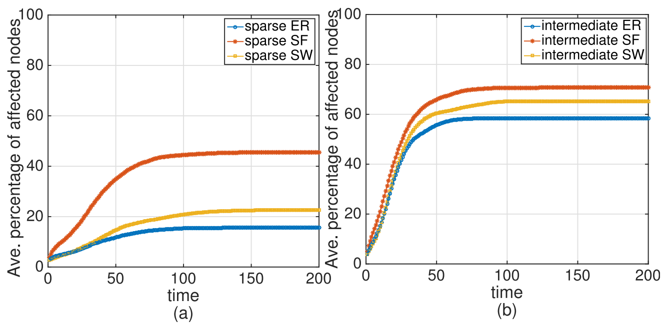

Fig. 6. Different network models: Propagation process over time, Average degree of nodes = 6, F_0 = 4 nodes of network X with max degree.

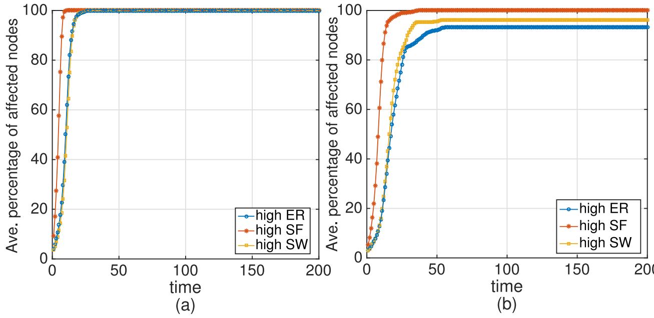

Fig. 7. Different network models: Propagation process over time, Average degree of nodes = 4, a) F_0 = 4 nodes of network X with max degree, b) F_0 = 4 random nodes of network X.

nodes of X with maximum degree, the Scale-Free network initially propagates the phenomenon faster than the others, but the difference with the Small World model is considerably lower. The Erdos-Renyi model is the slowest also in this case. Fig. 6 shows the same results for a higher average degree. We set the average node degree to 6, and we see that when the inter-connectivity between the two networks is sparse, the SF network propagates the phenomenon more than 2 times faster than SW and 3 times faster than ER. Also in case of intermediate inter-dependency, SF has the fastest propagation, but the difference between the models is less evident.

A similar set of experiments in shown in Fig. 7 for the case of high inter-connectivity, where we show that for both the choices of the spreader set, random or with maximum degree, the network models show a trend that is very similar to that in the previous experiments, only propagating the phenomenon faster.

We repeated these experiments for different configuration of inter-dependency, and network average degrees, and our results show that in most cases, the Scale-Free is the fastest network model in terms of propagating the phenomena to the whole network. The reason is that in the Scale-Free network model, as soon as high degree nodes are affected by the propagating phenomenon, there is a high chance that many of the nodes connected to them are also affected within a few time steps. This is because, the nodes that are connected to the hubs have small degree, while this is not the case for the other network models.

Also by comparing the results of sparse, intermediate and high inter-connectivity models, we conclude that by increasing the level of coupling between the two networks X, Y, the speed of propagation is also significantly increased and the three network models gets closer to each other. In fact, when the level of coupling between the two networks is sufficiently high, the three network models can be seen as almost homogeneous.

Moreover by comparing the results of three network models, and different models of initial spreaders selection, we conclude that the impact of the initial spreaders on the speed of phenomena propagation is more significant than the impact of network model. In other words, by properly selecting the initial spreaders in a slower network we can obtain a faster propagation than selecting random spreaders in networks that typically show a faster propagation.

For example, by looking at time $t = 80$ in Figs. 5a and 4a, we see that the average percentage of affected nodes for the Erdos-Renyi model with a designed set of initial spreaders

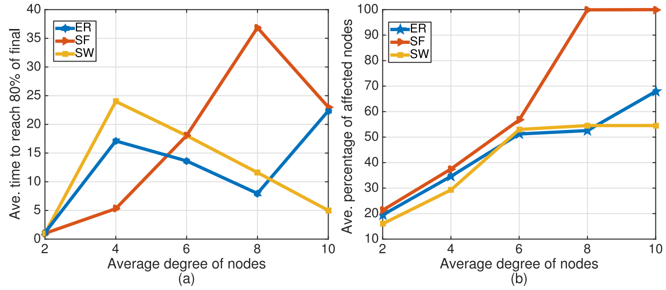

Fig. 8. Impact of average degree, (a) time to reach 80 percent of the final, (b) average percentage of the affected nodes at $T = 200$.

is 82 percent, while it is less than 80 percent for the Scale-Free and Small-World networks with a random choice of the initial spreader set.

7.2 On the Impact of the Average Degree

In this section, we focus on the role of the average degree (intra-degree) of the network graph on the speed of propagation. We recall from Section 6.1 that network models ER, SF and SW with varying average degree of nodes can be obtained by properly setting the parameters defining the network growth models. In order to investigate the impact of average degree independently of other parameters, we fix the inter-connectivity and intra-connectivity thresholds in terms of number of affected intra-neighbor or inter-parents (and not of their fraction with respect to total). We set $K_{\mathtt{xx}} = K_{\mathtt{yy}} = K_{\mathtt{xy}} = K_{\mathtt{yx}} = 2$. We select 30 random nodes as initial spreaders in network X and set $p_{\mathtt{max}}(\cdot) = 0.8$. Fig. 8 shows the results for the three network models under varying average degree.

In Fig. 8a we evaluate the time to reach 80 percent of the final propagation extent under varying average degree. We choose the 80 percent mark because this corresponds to the time at which, on average, the propagation process shows the maximum speed.

Fig. 8b shows the average percentage of nodes that will eventually be affected by the propagating phenomenon for 500 random seeds.

As we see from both Figs. 8a and 8b, increasing the average degree will speed up the propagation. In the case of SF, when the average degree goes from 2 to 8, Fig. 8a shows an increase in the time necessary for the propagation to reach the 80 percent mark. This increase in time is due to the related increase in the extent of propagation, as confirmed by Fig. 8b.

In contrast, when the degree goes from 8 to 10, the final number of affected nodes is constant (equal to 100 percent of the network nodes), and we see a decrease in the time to reach 80 percent of the final propagation extent. A similar trend is seen for ER and SW.

Whenever the number of affected nodes remains constant by changing the average degree, the time to reach 80 percent of the final propagation extent decreases. This confirms that increasing the average degree causes an increase in the propagation speed. The reason is that by increasing the intra-degree of the nodes, we also increase the probability that a sufficient number of neighbors of a node is affected. Therefore, nodes reach the threshold faster and as a result the speed of propagation increases.

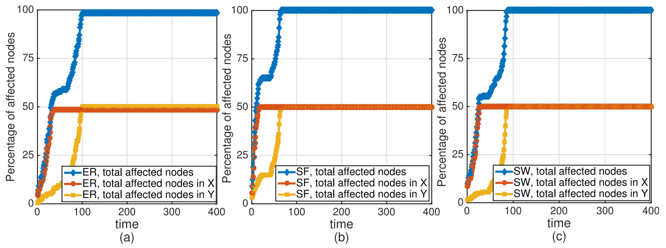

Fig. 9. Role of inter-connectivity nodes: Propagation process over time in the interdependent network and inside each network.

From Fig. 8b, we also notice that for any value of degree, the SF shows a faster propagation than the two other topologies, and is the only network which in this example achieves the 100 percent of propagation extent, while ER and SW reach only 69 and 55 percent, respectively.

7.3 On the Impact of the Inter-Connectivity Model

Now, we focus on the designed inter-connectivity models discussed in Section 6.2. We fix the topology of networks $G_{\mathtt{X}}$, $G_{\mathtt{Y}}$, and only change the selection of the borderline nodes. To motivate the importance of the nodes at the edge of interconnecting links, we first analyze a simple scenario of propagation. To show this we consider an interdependent network system with 200 nodes, with two networks of 100 nodes each. We assume that the set of initial spreaders consists of 10 random nodes of X. The coupling between the two networks is set to Min-Max scenario, where $\alpha = \beta = 6$. We investigate the percentage of nodes that are getting involved in the phenomena over time both in total, and for each network separately. The goal is to analyze the propagation process across the two networks. Fig. 9 shows the total number of involved nodes, and the number of involved nodes in each network for three different network models, Erdos-Renyi, Scale-Free, Small-World. We see that for the three network models, as soon a sufficient number of nodes in Y are involved, there is a significant increase in the speed of propagation. In fact, there is a point where the number of involved nodes in Y remains constant for a while, and after some time, it grows significantly and very fast. This explains the impact of cross network propagation on the overall speed of propagation.

Fig. 10 shows the propagation process over time, for four scenarios of designed inter-connectivity, and for a Scale-Free network model: (1) Max-Max, (2) Min-Min, (3) Max-Min, and (4) Min-Max.

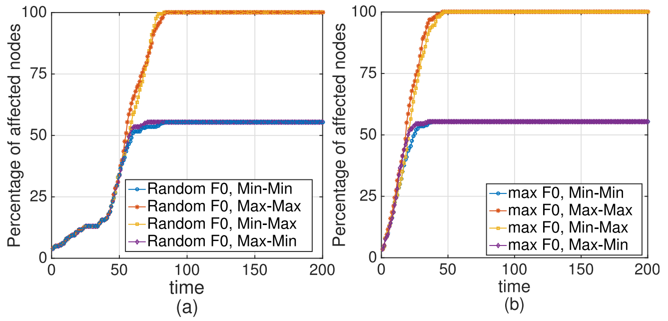

Fig. 11. Inter-connectivity models: Propagation process over time for Erdos-Renyi networks, Average degree of nodes = 4, (a) F_0 = 6 random nodes of network X, (b) F_0 = 4 nodes of X with max degree.

In this experiment, the average degree of the nodes in both networks is 4, $p_{\mathtt{max}_{\mathtt{X}}}(\cdot) = p_{\mathtt{max}_{\mathtt{Y}}}(\cdot) = 0.8$ and $p_{\mathtt{max}_{\mathtt{XY}}}(\cdot) = p_{\mathtt{max}_{\mathtt{YX}}}(\cdot) = 0.1$. We set $\alpha = 5$ and $\beta = 3$, meaning that five nodes of each network with max/min degree are selected, and each selected node of X is an inter-parent of three selected nodes in network Y and vice versa.

Fig. 10a represents the results for a scenario with six randomly selected initial spreaders in X. In contrast, Fig. 10b shows the same results for the case in which the four initial spreaders are the ones with maximum degree.

Notice that these results also confirm the conclusion that we made in Section 7.1. When the initial spreaders are selected as the ones with higher centrality, the propagation speed is considerably faster.

According to the results shown by Fig. 10, the Min-Min and Max-Min schemes have the lowest propagation speed. This is because the initial spreaders are located at network $G_{\mathtt{X}}$, where the phenomenon is generated. Since the borderline nodes of Y are those with minimum degree, the impact of the borderline nodes on other nodes in Y is lower. In fact, in this set of experiments, the phenomena gets stuck at the border of network Y, and cannot propagate further.

We also see that for the Max-Min scenario, the propagation is stopped, while for the Min-Max scenario it rapidly proceeds until all the nodes of the two networks are affected. Therefore, to ensure propagation in an interdependent network with Max-Min coupling model, the initial spreaders should be selected as the nodes with highest degree of network Y.

Figs. 11 and 12, show the results of a similar experiment for Erdos-Renyi and Small-World networks, respectively. To have a propagation speed comparable to the case of the Scale-Free model, we set $p_{\mathtt{max}_{\mathtt{XY}}}(\cdot) = p_{\mathtt{max}_{\mathtt{YX}}}(\cdot) = 0.8$. The

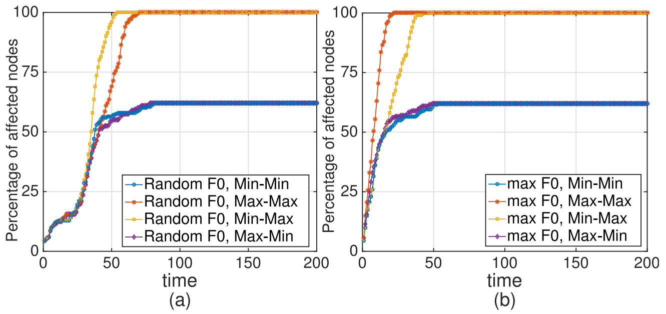

Fig. 10. Inter-connectivity models: Process over time for scale-free networks, Average degree of nodes = 4, (a) F_0 = 6 random nodes of network X, (b) F_0 = 4 nodes of X with max degree.

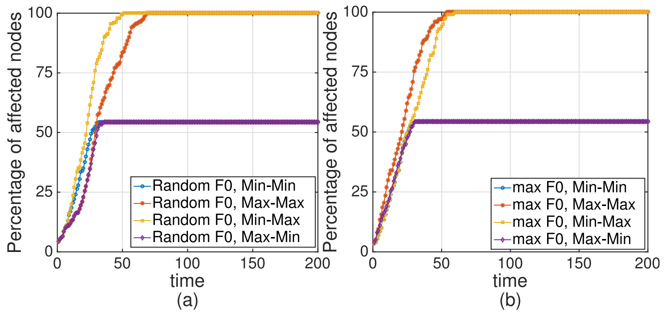

Fig. 12. Inter-connectivity models: Propagation process over time for Small-World networks, Average degree of nodes = 4, (a) F_0 = 6 random nodes of network X, (b) F_0 = 4 nodes of X with max degree.

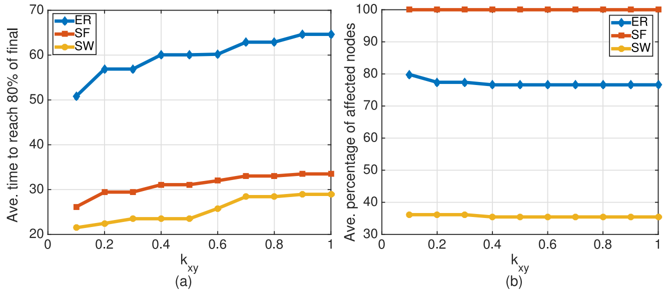

Fig. 13. Impact of inter-propagation threshold, (a) time to reach 80 percent of the final status, (b) average percentage of the affected nodes at $T = 200$.

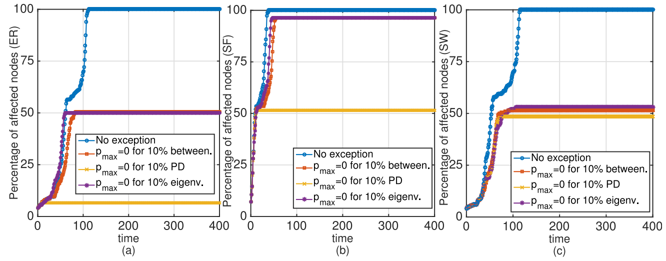

Fig. 14. Centrality: Comparison between different centrality metrics in terms of the impact on the propagation speed (a) ER, (b) SF, (c) SW.

results for these network models lead to the same conclusions already discussed for the case of Scale-Free.

By comparing the two scenarios in Figs. 11 and 12, we conclude that the Scale-Free network still provides faster propagation compared to the Erdos-Renyi network, even for the case in which the borderline nodes are less likely to fail, i.e., $p_{\mathtt{max}_{\mathtt{XY}}}, p_{\mathtt{max}_{\mathtt{YX}}}$ is smaller.

To conclude our discussion of this experimental scenario, for those networks in which the propagation is an unwanted event, the Min-Min inter-connectivity model is the best choice to slow down the propagation in both directions. Nevertheless it must be noted that by doing so, we may also slow down the diffusion of information, because we are forcing the degree of the inter-connectivity nodes to be low, and as a side effect we may increase the average length of the shortest paths. In some scenarios it may be useful to introduce two operational modes, devoted to address normal and abnormal circumstances. In normal circumstances the network could work in a Max-Max coupling mode, while in abnormal situations, the network should switch to the Min-Min coupling scheme.

In Fig. 13 we analyze the impact of the inter-connectivity threshold on the speed of propagation. As we did in Section 7.2, we consider the time to reach 80 percent of the final extent of network propagation in Fig. 13a, while Fig. 13b shows the average final percentage of affected nodes (in total for networks X and Y). Results are shown for $p_{\mathtt{max}}(\cdot) = 0.8, k_{xx} = 0.5, k_{yy} = 0.3$, average degree $= 8$ and for the designed Max-Max model of interconnectivity with $\alpha = \beta = 6$. Initially 20 random nodes are failed. We see that by increasing the inter-connectivity threshold, for three network topologies, the number of affected nodes is decreased, while the time to reach 80 percent of the final number of failed nodes is increased. This means that by increasing the threshold, the speed of propagation is decreased.

7.4 On the Impact of High Centrality Nodes

We devote this set of experiments to analyzing the capability of the new metric of centrality defined in Equation (6) to identify the most influential nodes in a propagation process.

To evaluate the influence of a node i on the phenomena propagation, we prevent it from being involved in the phenomena, and observe the propagation under this condition. In order to prevent node i from being affected, we set its propagation probability to 0, namely $p_{\mathtt{max}_{\mathtt{X}}}(i) = p_{\mathtt{max}_{\mathtt{YX}}}(i) = 0$. This means even if all of the parents of i are already affected, the probability that i is successively affected is zero. This implies that node i cannot propagate the phenomenon. If a node is not critical in propagating the phenomena, making it resistant may not lead to a significant change in the behavior of the propagation. By contrast, precluding an influential node from participating in the propagation process may have a considerable impact on the propagation speed.

We considered two networks $G_{\mathtt{X}}$ and $G_{\mathtt{Y}}$, with $n_{\mathtt{x}} = n_{\mathtt{y}} = 100$. We sorted the nodes on the basis of the centrality metric and we determined the 5 percent of the nodes with highest centrality in each network. We performed this experiment for three different centrality metrics: 1) betweenness centrality, 2) eigenvector centrality, and 3) the PD centrality metric that is given in Equation (6). For these nodes we set $p_{\mathtt{max}_{\mathtt{X}}} = p_{\mathtt{max}_{\mathtt{YX}}} = 0$. For all the other nodes of the network, we set $p_{\mathtt{max}_{\mathtt{X}}} = p_{\mathtt{max}_{\mathtt{Y}}} = p_{\mathtt{max}_{\mathtt{XY}}} = p_{\mathtt{max}_{\mathtt{YX}}} = 0.8$.

A set of four random nodes in X is selected as the set of initial spreaders, and the propagation of the phenomena over time through the two networks is observed. Fig. 14 shows the evolution of the phenomena propagation over time, for the three types of networks, and for the three types of centrality compared to the case with no exceptional nodes, where all the nodes have the same propagation probability. For all cases shown in Fig. 14, by making the nodes with high centrality resistant to the phenomena propagation, the speed of propagation is significantly reduced. The behavior of phenomena propagation is different depending on which metric of centrality is used to select the resistant nodes. We see that the proposed centrality metric performs better than the other metrics in terms of determining the influential nodes, since the percentage of the affected nodes after 400 time steps is much less when we adopt the new centrality metric than with the other standard metrics. In the case of the Small-World network, the difference between the proposed metric and the standard metrics is not as large. The reasons for this are the following: 1) the network is more uniform in terms of the node degrees compared to the SF and ER networks; 2) according to the SW model, most of the nodes can be reached from every other node in a small number of hops and therefore, the difference between shortest paths and all other paths is not significant. Therefore, in view of the node uniformity, there is a high chance that the new and the classic centrality metrics will select the same nodes.

In the case of an ER network using the PD centrality, by making only 5 percent of the nodes resistant to the phenomena propagation, we are able to stop the propagation of the phenomena from network X to network Y when less than 10 percent of the nodes are affected. In the case of the Scale-Free network, using the PD

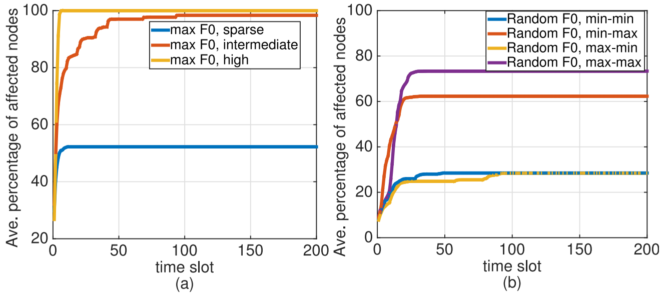

Fig. 15. Impact of inter-connectivity model, (a) randomized interconnectivity for CAIDA, (b) designed interconnectivity for CAIDA.

centrality, the phenomena propagates from X to Y but it stops very soon thereafter, i.e. the percentage of affected nodes remains constant after 52 percent of the nodes are involved. In contrast, using betweenness and eigenvector centrality metrics, the phenomena almost fully propagates except for the 5 percent of the nodes with exceptional resistance, as can be seen in Fig. 14b. We performed similar experiments for many other scenarios and other values of $p_{\max}(\cdot)$. All our results confirm that the proposed metric can be successfully used to identify the influential nodes in the context of phenomena propagation.

7.5 On the Impact of Interconnectivity Model and Initial Spreaders in Real-World Network

We now consider the case of real network topologies and study the impact of interconnectivity model and of the criterion used to select the initial spreaders on the speed of propagation. To represent networks X and Y we use the CAIDA [35] networks AS28583 and AS10024, with 284 and 318 nodes, respectively. We model the inter-connectivity between X and Y in two ways: randomized and designed. We set $k_{xx} = k_{yy} = 0.3, k_{xy} = k_{yx} = 0.4$. For Fig. 15a, $|F_0| = 30$ nodes of network X with highest degree, and for Fig. 15b, $|F_0| = 30$ random nodes of X. For the randomized interconnectivity model, we set $p_{\max_X} = p_{\max_Y} = 0.8, P_{XY} = 0.025, P_{YX} = 0.028, \alpha_s = \beta_s = 2, \alpha_i = \beta_i = 8, \alpha = 284, \beta = 318$, and for the designed model, we set $\alpha = \beta = 8$. The average degree of network X is 2.92 and that of network Y is 2.11.

Fig. 15 shows the average percentage of affected nodes over time. The results are averaged over 500 runs in which we vary the random choice of border nodes of the randomized inter-connectivity model and the random selection of the initial spreaders of the designed model. As expected, we see that in the case of randomized inter-connectivity, the high inter-connectivity model spreads phenomena faster, and in the case of the designed model, the Max-Max interconnectivity provides faster propagation with respect to the three other models. We also see that for the same number of inter-connection links, intermediate inter-connectivity model with high degree nodes as initial spreaders is faster when compared to the designed model with random initial spreaders. In the first case, a full propagation is achieved while in the second case, the phenomena propagation only reaches 77 percent of the network. This confirms the fact that for similar levels of inter-connectivity, the choice of the initial spreaders has more impact than the inter-connectivity model in influencing the propagation process.

8 Network Design Revisited

In Section 7 we investigated the propagation of phenomena for interdependent networks under different scenarios. We performed an extensive set of experiments to evaluate the impact of network characteristics, such as network structure, coupling model, selection of initial spreaders, and so on, on the speed of propagation. Our work provides insights toward designing efficient interdependent networks, confirming the discussion given in Section 5. From a network design perspective, we can summarize our observations as follows:

(a) Under the same condition for the coupling and initial spreaders, the SF networks provide a faster propagation speed with respect to the ER and SW, and the ER has the slowest propagation on average.
(b) Under the same coupling scenario, the impact of the selection of initial spreaders on the speed of propagation is more significant than the network model. To this purpose we proposed a new metric of centrality, called path-degree centrality, that is more accurate than classic centrality metrics in finding the most influential nodes in the propagation. Thus by having the path-degree central nodes as initial spreaders in a slow network such as the ER we will have a faster propagation than choosing random nodes in SF or SW networks.
(c) By increasing the level of coupling between two networks, the speed of phenomena propagation increases. Coupling the nodes with highest centrality of each network (e.g., degree, betweenness, path-degree, etc.) increases the speed of propagation. By contrast, coupling the nodes with minimum centrality decreases the propagation speed.

9 Conclusion

In this paper, we proposed an analytical framework that enables us to study the propagation of phenomena in a general interdependent network and to derive useful guidelines to design an efficient interconnected network system.

We performed an extensive experimentation to evaluate the impact on the propagation speed of features such as network structure, inter-network connectivity and choice of initial spreaders. Furthermore, we proposed a new centrality metric that is more powerful than the standard centrality metrics in determining the most influential nodes for the propagation of a phenomenon across interdependent networks. Experiments show that our metric successfully determines the nodes that are more influential than others in determining the propagation speed. We showed that the propagation speed can be slowed down, or increased by making the high centrality nodes more resistant or more propense to propagate the phenomenon.

As a future work we plan to investigate similar propagation models in the case of dynamic networks, where the topology of the networks changes over time.

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References

[1] M. Parandehgheibi, E. Modiano, and D. Hay, "Mitigating cascading failures in interdependent power grids and communication networks," in *Proc. IEEE Int. Conf. Smart Grid Commun.*, Nov. 2014, pp. 242–247.

[2] J. Gao, S. V. Buldyrev, H. E. Stanley, and S. Havlin, "Networks formed from interdependent networks," *Nature Physics*, vol. 8, pp. 40–48, Dec. 2011.

[3] S. V. Buldyrev, R. Parshani, G. Paul, H. E. Stanley, and S. Havlin, "Catastrophic cascade of failures in interdependent networks," *Nature*, vol. 464, pp. 1025–1028, 2010.

[4] S. V. Buldyrev, N. W. Shere, and G. A. Cwilich, "Interdependent networks with identical degrees of mutually dependent nodes," *Physical Rev. E*, vol. 83, 2011, Art. no. 016112.

[5] R. Parshani, S. V. Buldyrev, and S. Havlin, "Interdependent networks: Reducing the coupling strength leads to a change from a first to second order percolation transition," *Physical Rev. Lett.*, vol. 105, 2010, Art. no. 048701.

[6] J. Shao, S. V. Buldyrev, S. Havlin, and H. E. Stanley, "Cascade of failures in coupled network systems with multiple support-dependent relations," *Physical Rev. E*, vol. 83, 2011, Art. no. 036116.

[7] C. D. Brummitt, R. M. D'Souza, and E. A. Leicht, "Suppressing cascades of load in interdependent networks," *Proc. Nat. Academy Sci. United States America*, vol. 109, no. 12, pp. E680–E689, 2012.

[8] R. M. D'Souza and J. Nagler, "Anomalous critical and supercritical phenomena in explosive percolation," *Nature Physics*, vol. 11, pp. 531–538, 2015.

[9] J. Leskovec, L. Adamic, and B. Huberman, "The dynamics of viral marketing," *ACM Trans. Web*, vol. 1, no. 1, 2007, Art. no. 5.

[10] D. Kempe, J. Kleinberg, and E. Tardos, "Maximizing the spread of influence through a social network," in *Proc. 9th ACM SIGKDD Int. Conf. Knowl. Discovery Data Mining*, 2003, pp. 137–146.

[11] D. Kempe, J. Kleinberg, and E. Tardos, "Influential nodes in a diffusion model for social networks," in *Proc. 32nd Int. Colloquium Automata Languages Programming*, 2005, pp. 1127–1138.

[12] L. Blume, D. Easley, J. Kleinberg, R. Kleinberg, and E. Tardos, "Which networks are least susceptible to cascading failures?" in *Proc. IEEE 52nd Annu. Symp. Found. Comput. Sci.*, Oct. 2011, pp. 393–402.

[13] O. Yagan, D. Qian, J. Zhang, and D. Cochran, "Optimal allocation of interconnecting links in cyber-physical systems: Interdependence, cascading failures, and robustness," *IEEE Trans. Parallel Distrib. Syst.*, vol. 23, no. 9, pp. 1708–1720, Sep. 2012.

[14] K. Lee, C. D. Brummitt, and K. I. Goh, "Threshold cascades with response heterogeneity in multiplex networks," *Physical Rev. E*, vol. 90, 2014, Art. no. 062816.

[15] C. D. Brummitt, K. Lee, and K. I. Goh, "Multiplexity-facilitated cascades in networks," *Physical Rev. E*, vol. 85, 2012, Art. no. 045102.

[16] O. Yagan and V. Gligor, "Analysis of complex contagions in random multiplex networks," *Physical Rev. E*, vol. 86, 2012, Art. no. 036103.

[17] J. Chalupa, P. L. Leath, and G. R. Reich, "Bootstrap percolation on a Bethe lattice," *J. Physics C*, vol. 12, 1979, Art. no. L31.

[18] G. J. Baxter, S. N. Dorogovtsev, A. V. Goltsev, and J. F. F. Mendes, "Bootstrap percolation on complex networks," *Physical Rev. E*, vol. 82, 2010, Art. no. 011103.

[19] C. M. Schneider, N. Yazdani, N. A. M. Araujo, S. Havlin, and H. J. Herrmann, "Towards designing robust coupled networks," *Scientific Rep.*, vol. 3, Jun. 2013, Art. no. 1969.

[20] D. J. Watts, "A simple model of global cascades on random networks," *Proc. Nat. Academy Sci. United States America*, vol. 99, pp. 5766–5771, 2002.

[21] M. Kitsak, et al., "Identification of influential spreaders in complex networks," *Nature Physics*, vol. 6, pp. 888–893, Aug. 2010.

[22] P. Hines, E. Cotilla-Sanchez, and S. Blumsack, "Do topological models provide good information about electricity infrastructure vulnerability?" *Chaos Amer. Inst. Physics*, vol. 20, no. 3, 2010, Art. no. 033122.

[23] P. Dodds and D. Watts, "Universal behavior in a generalized model of contagion," *Physics Rev. Lett.*, vol. 92. 2004, Art. no. 218701.

[24] F. Allen and D. M. Gale, "Financial contagion," *J. Political Economy*, vol. 108, pp. 1–33, Feb. 2000.

[25] R. M. Anderson and R. M. May, *Infectious Diseases of Humans*. Oxford, U.K.: Oxford Univ. Press, 1992.

[26] C. M. Grinstead and L. J. Snell, *Introduction to Probability*. Providence, RI, USA: Amer. Math. Soc., Jul. 1997.

[27] L. Freeman, "A set of measures of centrality based on betweenness," *Sociometry*, vol. 40, pp. 35–41, 1977.

[28] P. R. Gould, "On the geographical interpretation of eigenvalues," *Trans. Inst. British Geographers*, vol. 42, pp. 53–86, Dec. 1967.

[29] G. Sabidussi, "The centrality index of a graph," *Psychometrika*, vol. 31, pp. 581–603, 1966.

[30] MATLAB. (2015). [Online]. Available: http://www.mathworks.com/products/matlab/

[31] W. Cho, K. I. Goh, and I. M. Kim, "Correlated couplings and robustness of coupled networks," 2010. [Online]. Available: http://arxiv.org/abs/1010.4971

[32] A. L. Barabasi and R. Albert, "Emergence of scaling in random networks," *Science*, vol. 286, pp. 509–512, 1999.

[33] D. J. Watts and S. H. Strogatz, "Collective dynamics of small-world' networks," *Nature*, vol. 393, pp. 440–442, 1998.

[34] P. Erdos and A. Renyi, "On random graphs," *Publicationes Mathematicae*, vol. 6, pp. 290–297, 1959.

[35] The cooperative association for internet data analysis (CAIDA), "Macroscopic Internet Topology Data Kit (ITDK)," 2013. [Online]. Available: http://www.caida.org/data/active/ internet-topology-data-kit/

Hana Khamfroush received the PhD degree with honors in telecommunications engineering from the University of Porto, Portugal, in 2014. She is now a research associate in the Computer Science Department, Penn State University. She was named a rising star in EECS by MIT in 2015. She was served as TPC member and reviewer of several international conferences and Journals. She is currently the social media co-chair of the IEEE N2Women community.

Novella Bartolini received the graduate degree with honors and the PhD degree in computer engineering from the University of Rome, Italy, in 1997 and 2001, respectively. She is now associate professor with Sapienza University of Rome and visiting professor with Penn State University, since 2014. She was program chair and program committee member of several international conferences. She has served on the editorial board of Elsevier the *Computer Networks* and the *ACM Wireless Networks*.

Thomas F. La Porta is the Wlliam E Leonhard chair professor in the Department of Computer Science and Engineering, Penn State University. He is the director of the Institute for Networking and Security Research. Prior to joining Penn State, he was with Bell Laboratories, where he was director of the Mobile Networking Research Department. He received the Bell Labs Distinguished Technical Staff Award, and an Eta Kappa Nu Outstanding Young Electrical Engineer Award in 1996. He also won Thomas Alva Edison Patent Awards in 2005 and 2009. He was the founding editor-in-chief of the *IEEE Transactions on Mobile Computing*, and served as editor-in-chief of the *IEEE Personal Communications Magazine*. He is a fellow of the IEEE and the Bell Labs.

Ananthram Swami received the BTech degree from IIT-Bombay, the MS degree from Rice University, and the PhD degree from USC, all in electrical engineering. He is with the US Army Research Laboratory as the Army's senior research scientist. Prior to this, he held positions with Unocal Corporation, USC, CS-3 and Malgudi Systems. He was a statistical consultant to the California Lottery, developed a Matlab toolbox for non-Gaussian signal processing. He has held visiting faculty positions at INP, Toulouse., and Imperial College. His work is in the broad area of network science, with emphasis on wireless communication networks. He is a fellow of an ARL and the IEEE.

Justin Dillman is working toward the undergraduate degree at Pennsylvania State University. He is currently studying computer science and performs research with the CSE Department.