Outsourced Pattern Matching

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Joint work with Sebastian Faust and Carmit Hazay





 Outsourcing computation of a public function f to a possibly malicious cloud provider (a.k.a. the server)

Weak client





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"Compute f(x) for me!"













 Outsourcing computation of a public function f to a possibly malicious cloud provider (a.k.a. the server)



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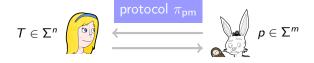
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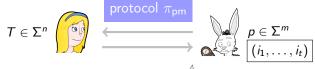


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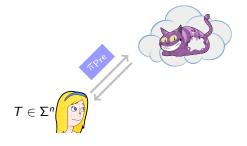








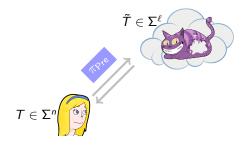










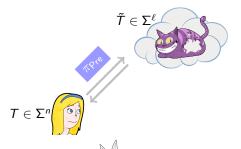






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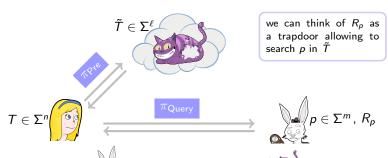




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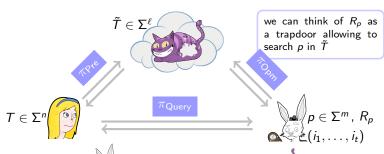




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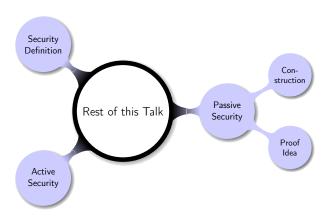


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- An extension achieving active security (see the paper)

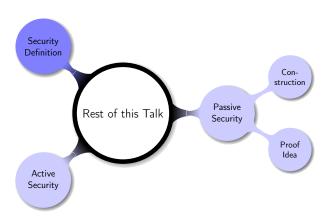


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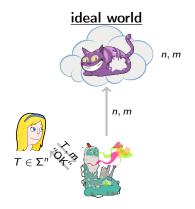
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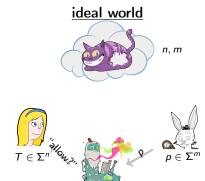




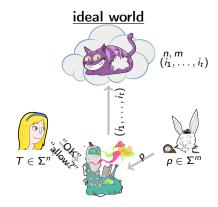




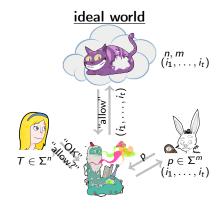




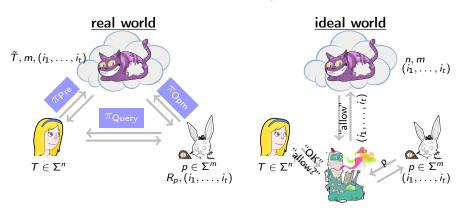




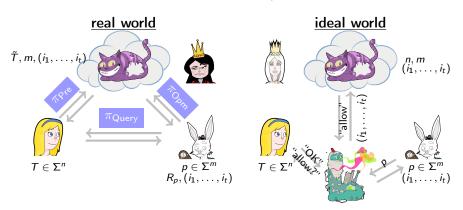




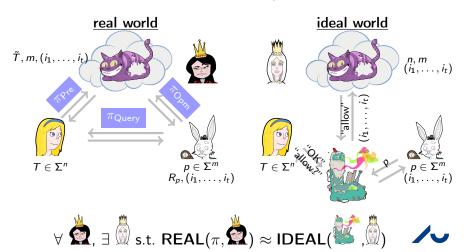










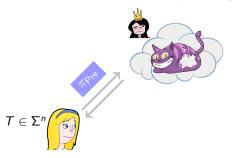






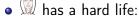




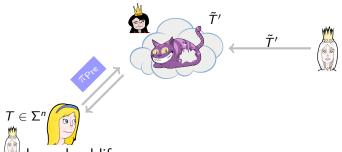






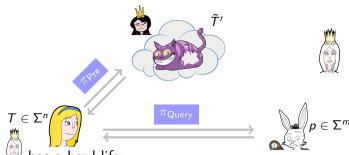






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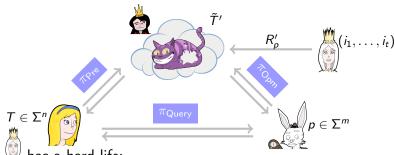




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$$(\tilde{T}',R_p')\approx (\tilde{T},R_p)$$



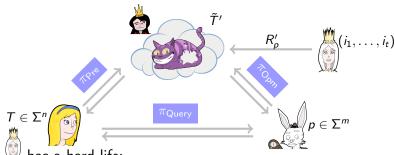


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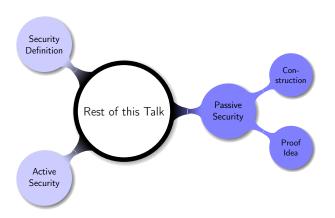
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However p was not known when \tilde{T}' has been computed!

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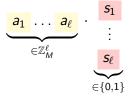


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$$\underbrace{a_1} \cdots \underbrace{a_\ell}_{\in \mathbb{Z}_M^\ell}$$

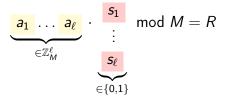


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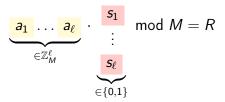


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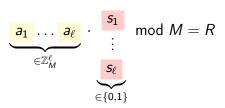
Goal

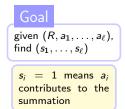
given (R, a_1, \ldots, a_ℓ) , find (s_1, \ldots, s_ℓ)

 $s_i = 1$ means a_i contributes to the summation



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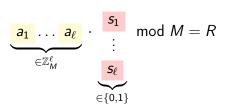


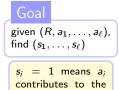


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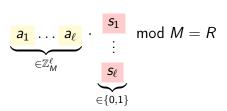


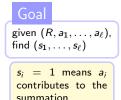
summation

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(In practice compute $R_p = f(\kappa, p)$ for PRF f and store only κ .)

- Protocol π_{Query} : Any two-party protocol for oblivious evaluation of $f(\kappa, p)$
- Protocol π_{Opm} : gives R_p to and the latter solves subset sum instance (R_p, \tilde{T})

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 - Even $\lambda=10^4$ (i.e., subset sum elements of size ≈ 10 KByte) allows to process texts of less than 100 bits
- To overcome the above problems, we define an extension of the previous solution based on packaging



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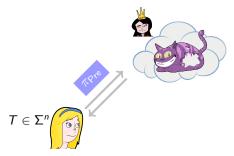
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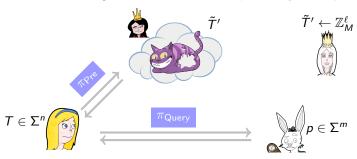








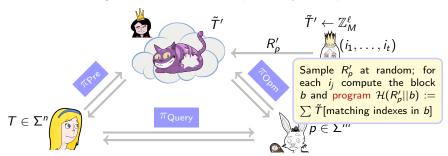






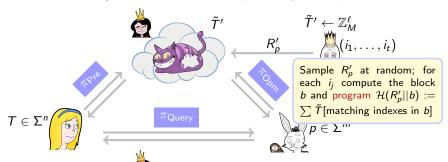






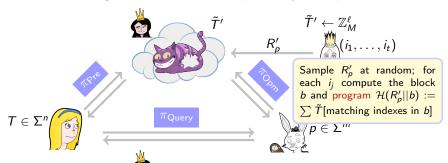


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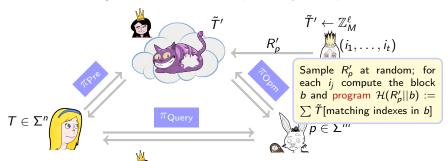
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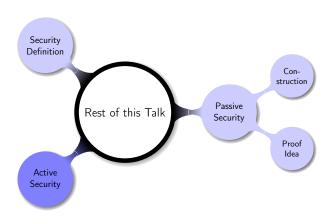
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 - HYB $(\pi, 2) \approx_s IDEAL(3)$ (programming can fail)

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 - This requires expensive cut-and-choose techniques, which we avoid by a smart "on-the-fly" verification trick

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 - Extensions (pattern matching with wildcards, approximate pattern matching, hiding the length of the text/pattern)

Thank you!



