On the Non-Malleability of the Fiat-Shamir Transform

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TECHNISCHE UNIVERSITÄT DARMSTADT

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Our result in a nutshell

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- Prove Fiat–Shamir NIZKs to be simulation-sound and -extractable under mild requirements
- Corollary (of known applications of NIZKs):
  - efficient leakage-resilient CCA2-secure PKE
  - efficient KDM CCA2-secure PKE
  - efficient leakage-resilient signatures

Outline

1. Established notions and known results
   - Interactive protocols
   - Non-interactive protocols
   - Non-malleability for NIZKs

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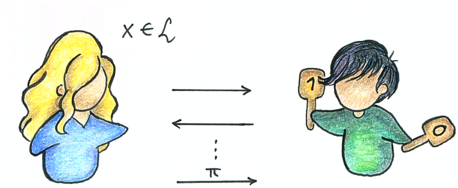
2. Our contribution
    - Properties of the Fiat–Shamir transform
    - Applications

Interactive proofs (IP)



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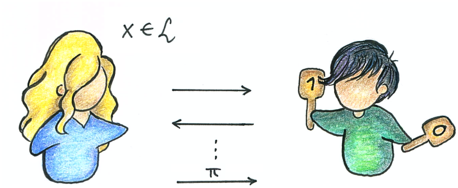

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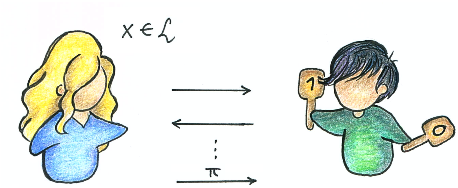

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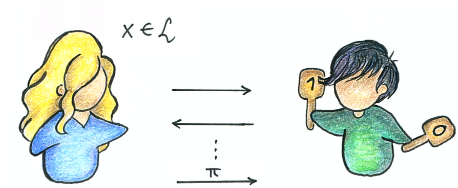

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Completeness + Soundness

Zero knowledge

In a ***zero-knowledge*** proof, $\mathcal{P}$ convinces $\mathcal{V}$ that a statement is true, but $\mathcal{V}$ does not learn anything beyond its validity

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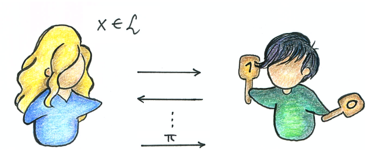

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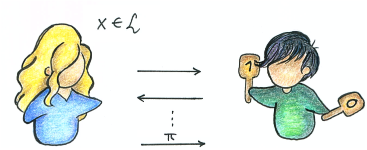

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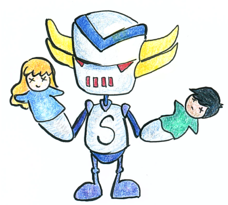
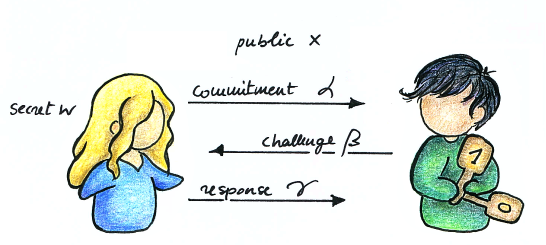


...can be **simulated** by efficient algorithm $\mathcal{S}$

Sigma protocols

- $\mathcal{P}$ and $\mathcal{V}$ share input x
- $\mathcal{P}$ holds private input w (w wintess for $x \in \mathcal{L}$)
- 3-move interaction
  1. commitment
  2. challenge
  3. response



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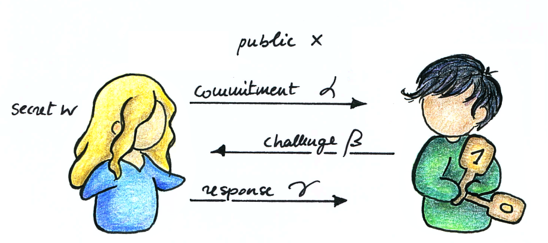

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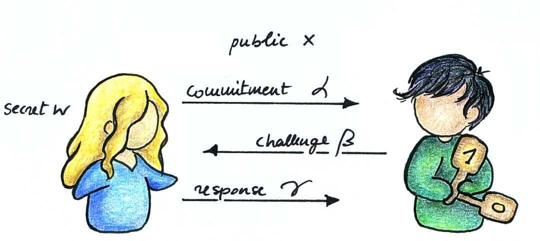

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- **Special soundness**
  Exists efficient extractor $\mathcal{E}_{sp}$ that outputs witness given two different accepting proof with same α

Fiat–Shamir transform

How to prove in zero knowledge without interaction

Non-interactive proofs

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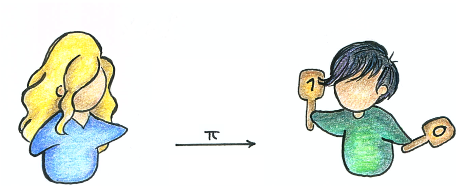

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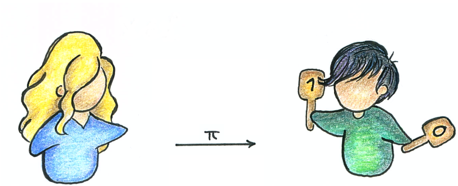

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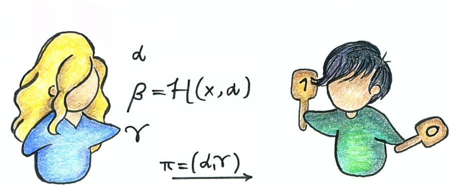

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- Turns 3-move IP into **non-interactive** ones
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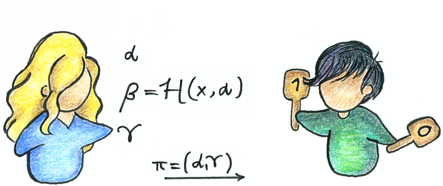

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> The Fiat–Shamir transform turns any Sigma protocol Σ into a *non-interactive zero-knowledge* protocol Σ_{FS}

Simulation soundness

Soundness not sufficiently strong for NIZKs

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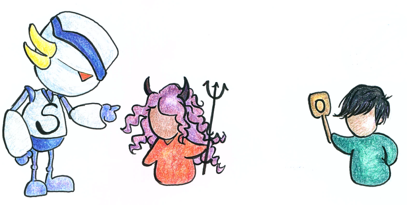

A NIZK is ***simulation-sound*** if no $\check{\mathcal{P}}$ can produce fresh accepting proofs of false statements, even if she observes simulated (fake) proofs

Simulation extractability

Knowledge extraction for IP

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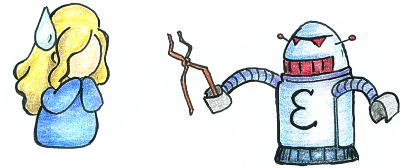

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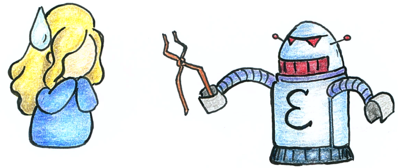

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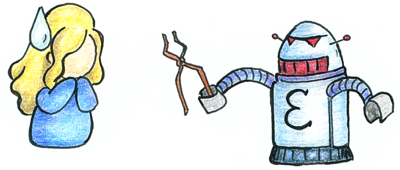

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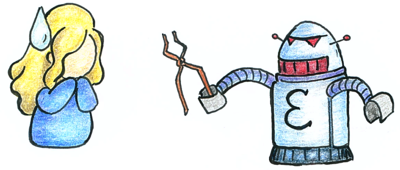

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Strong simulation-extractability
$\mathcal{E}$ on-line extractor (does not rewind $\mathcal{P}$)

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   - Properties of the Fiat–Shamir transform
   - Applications

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* A 3-move protocol has ***quasi-unique responses*** if it is hard to find two valid proofs which differ only in γ

Applications in Cryptography

Simulation-sound and extractable NIZKs as building-blocks:

- from CPA to CCA security for public-key encryption
- Key-dependent message (KDM) security
- Leakage-resilient signatures

Chosen-ciphertext security

Twin encryption and zero-knowledge proof

Naor–Yung transformation

- start from PKE scheme
- encrypt message twice under two independent public keys
- add proof of equality of plaintexts
  (witness $=$ message $+$ randomness used by Enc)

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Our concrete instantiation:

- LR-CPA PKE scheme, generalization of ElGamal [BHHO09]
- NIZK protocol Σ_{FS} derived from sigma protocol associated with the corresponding NY language

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A PKE scheme has *key-dependent message* security if it remains secure even given the encryption of some (known) functions of the decryption key

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  * Weak simulation extractability guarantees that $\mathcal{E}$ extracts w with non-negligible probability

Summary

	CRS	RO
NIZK	✓	✓
Simulation soundness	✓	✓
Weak simulation extractability	✓	✓
Full simulation extractability	✓	?

Our contribution:

- Formalized security properties for NIZKs in the RO model
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Open Problem:

Can we achieve **full simulation extractability**?

Thank you for your attention

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