On the Connection between Leakage Tolerance and Adaptive Security

Daniele Venturi

Aarhus University

PKC 2013—Nara

Joint work with Jesper Buus Nielsen and Angela Zottarel

Secure Message Transmission

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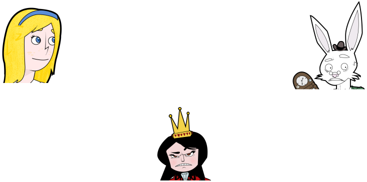

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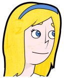

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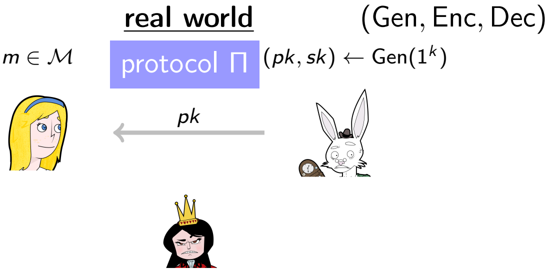

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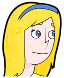

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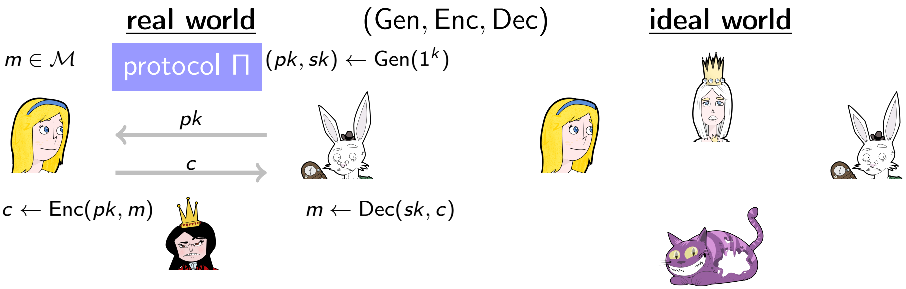

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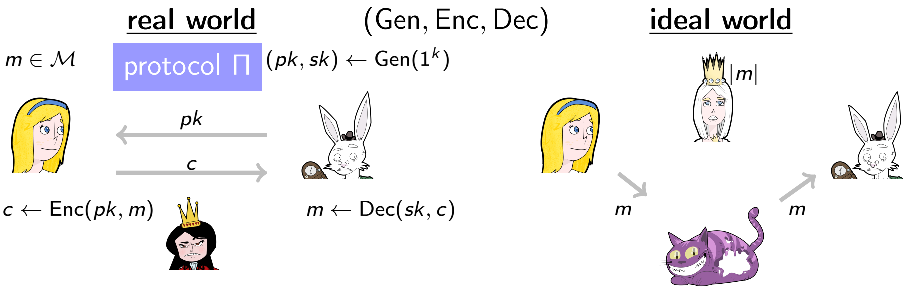

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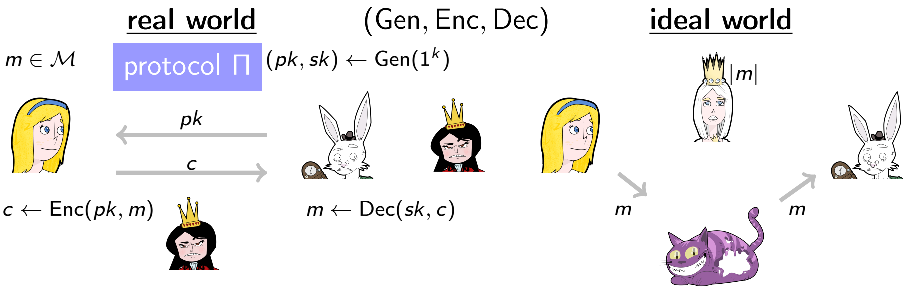

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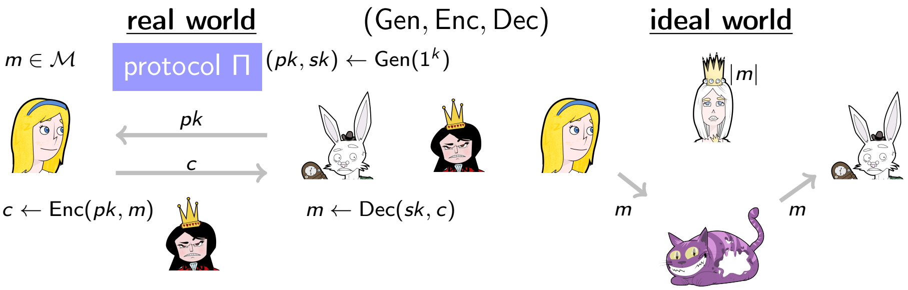

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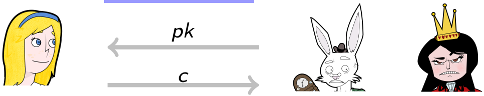

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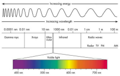

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- Even partial leakage on σ_A or σ_B sufficient to put security of the scheme under attack on edge

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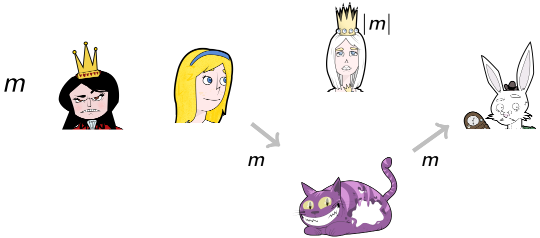

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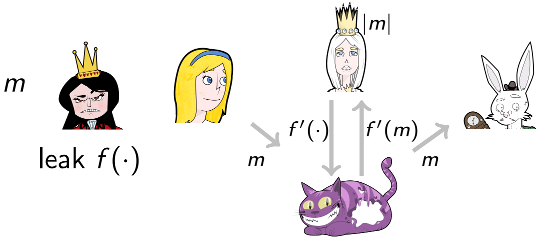

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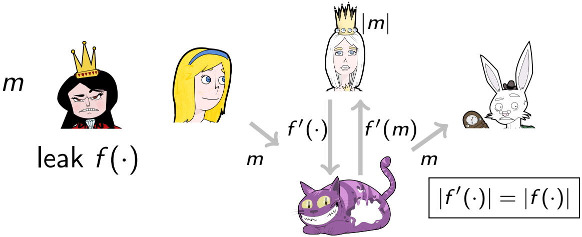

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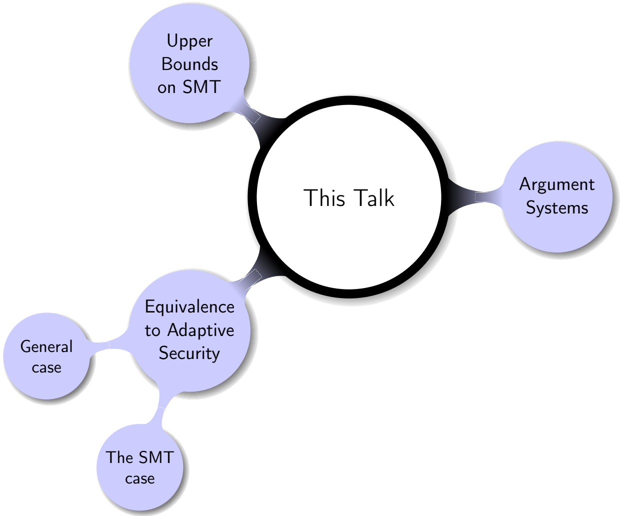
- <u>This work:</u> We look at the other direction

Roadmap



Upper Bounds on SMT
This Talk
Argument Systems
Equivalence to Adaptive Security
General case
The SMT case

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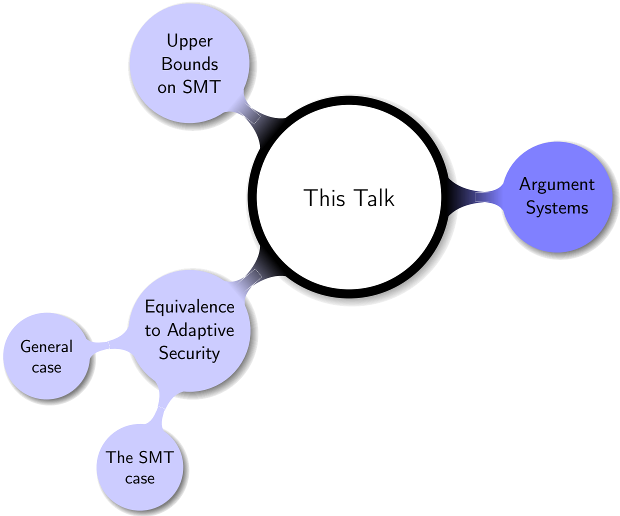
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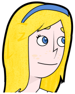

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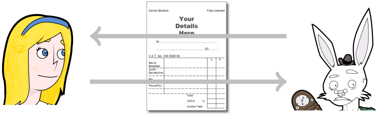

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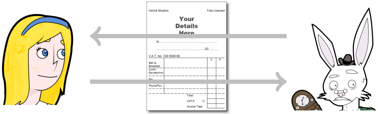

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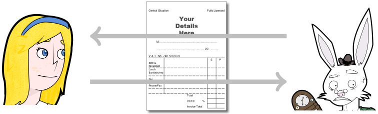

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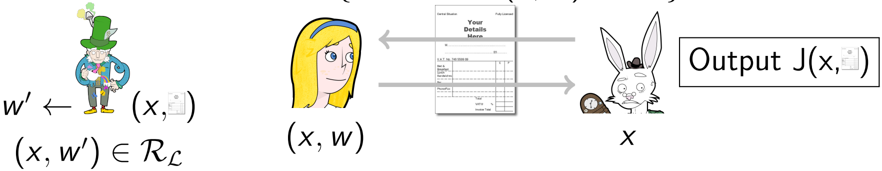

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  - Main ingredients: PCP theorem + Merkle trees

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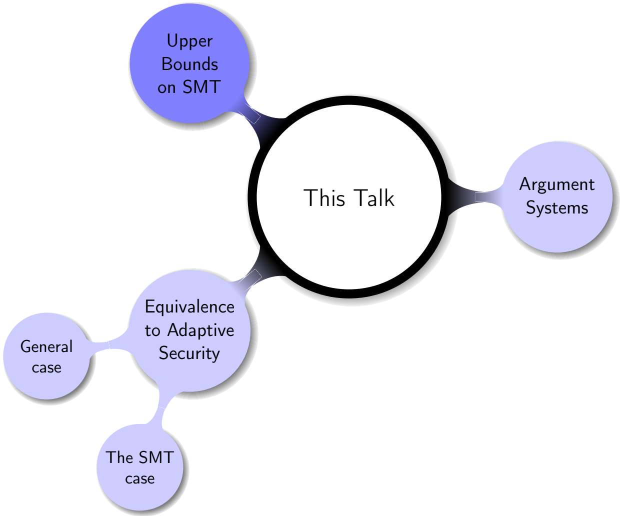

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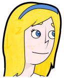
Leakage-tolerant SMT requires large keys

Theorem: Assume collision resistant function ensembles exist. Let Π be a leakage tolerant protocol for SMT tolerating poly-logarithmic leakage. Then,

$$|\mathcal{SK}| \geq (1-\epsilon)|\mathcal{M}| \text{ for negligible } \epsilon.$$

Sketch of the proof



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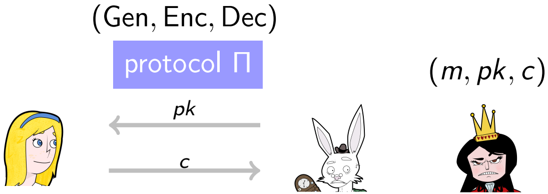

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$$\mathcal{L} = \{x := (m, pk, c) : \exists w = (sk, r_G) \text{ "explaining" } x\}$$

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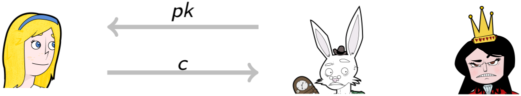

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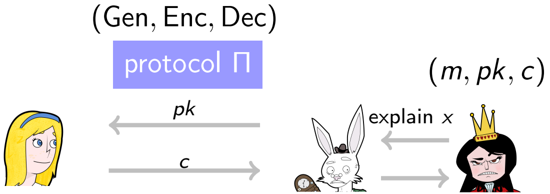

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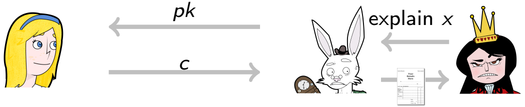

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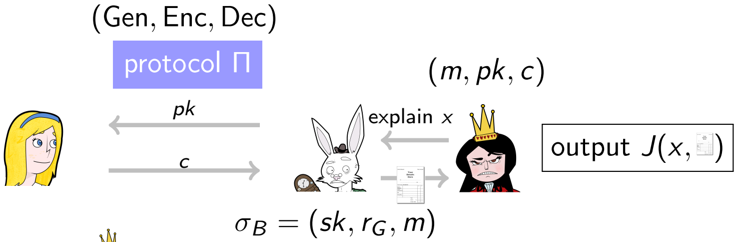

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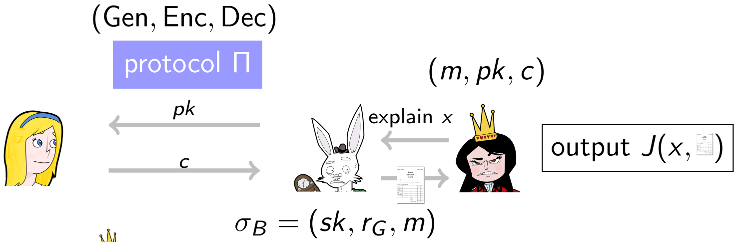

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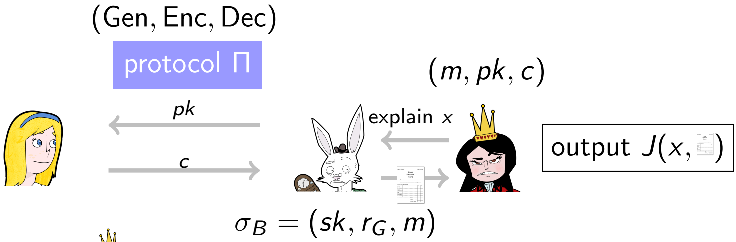

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  -  can compute the verifier's next message and "hard-wire" the result in the next leakage query

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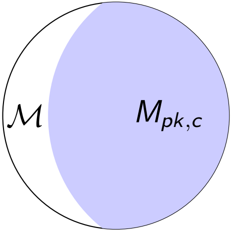

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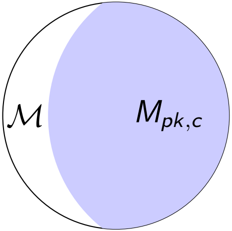

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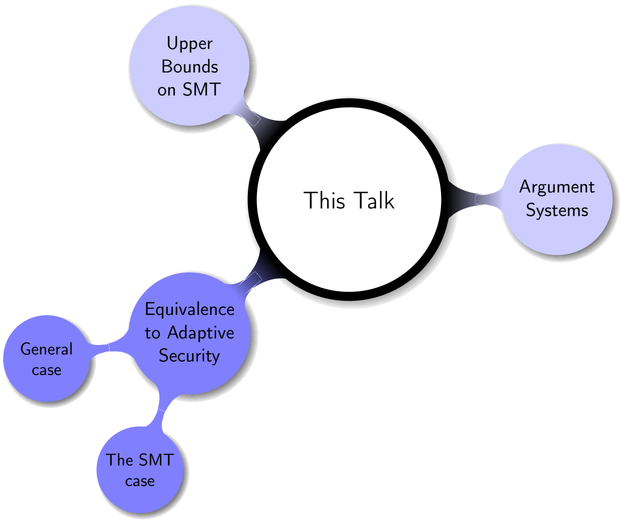
$$\forall m_0 \neq m_1 \in M_{pk,c} \text{ it is } sk_0 \neq sk_1 \Rightarrow |\mathcal{SK}| \geq |M_{pk,c}|$$

Roadmap



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- From this we get a valid simulator  in the ideal world
- Since the proof will accept with overwhelming probability, a simulator  for the adaptive security game can run  and extract from it a consistent state $w = (sk, r_G)$

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  - If we now extract w from the proof in (2) above, we get the right distribution (unless collision resistance is broken)

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- We obtain in this case a weaker form of adaptive security, i.e. we can still extract a consistent internal state but this may not be indistinguishable from a real state

Conclusions

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- We have shown that for some corruption case and for poly-logarithmic leakage
    - SMT requires a key as long as the message being encrypted
    - Leakage tolerance implies adaptive security

Thank You!

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