

Monday, December 16

9:30 - 10:30	Bruce Reed - How many edges force an H -minor?
10:30 - 11:00	Coffee Break
11:00 - 11:20	Chun-Hung Liu - Well-quasi-ordering graphs by the topological minor relation
11:25 - 11:45	Jean-Florent Raymond - An edge variant of Erdős-Pósa property
11:50 - 12:10	Irene Muzi - Subdivisions in 4-connected graphs of large tree-width
12:10 - 3:30	Lunch and discussion
3:30 - 4:30	Dan Kral - FO limits of trees
4:30 - 5:00	Coffee Break
5:00 - 5:20	Louis Esperet - Coloring planar graphs with three colors and no large monochromatic components
5:25 - 5:45	Kenta Ozeki - An extension to 3-colorable or Eulerian triangulations
5:50 - 6:10	Matej Stehlik - Coloring higher dimensional projective quadrangulations

Tuesday, December 17

9:30 - 10:30	Julia Chuzhoy - Polynomial bounds for the Grid-Minor Theorem
10:30 - 11:00	Coffee Break
11:00 - 11:20	Marcin Pilipczuk - Network sparsification for Steiner problems on planar and bounded-genus graphs
11:25 - 11:45	Nicolas Trotignon - The stable set problem is FPT in bull-free graphs
11:50 - 12:10	Michal Pilipczuk - Minimum bisection is fixed-parameter tractable
12:10 - 3:30	Lunch and discussion
3:30 - 4:30	Daniel Marx - Everything you always wanted to know about the parameterized complexity of Subgraph Isomorphism (but were afraid to ask)
4:35 - 5:05	Coffee Break
5:05 -	Problem session

Wednesday, December 18

9:30 - 10:30	Paul Seymour - Disjoint Dijoins
10:30 - 11:00	Coffee Break
11:00 - 11:20	Reinhard Diestel - Tangles, brambles, blocks, and profiles
11:25 - 11:45	Blair Sullivan - To be announced
11:50 - 12:10	Anita Liebenau - On the oriented cycle game
12:15 - 12:35	Tomas Kaiser - Hamilton connectivity and the scattering number of interval graphs
12:30 -	Lunch and excursion

Thursday, December 19

9:30 - 10:30	Stephan Thomasse - Cliques and stable sets in proper graph classes
10:30 - 11:00	Coffee Break
11:00 - 11:20	Frederic Maffray - Wheel and antiwheel-free graphs
11:25 - 11:45	Irena Penev - Isolating induced subgraphs
11:50 - 12:10	Aurelie Lagoutte - Clique stable separation in perfect graphs with no balanced skew partitions
12:10 - 3:30	Lunch and discussion
3:30 - 3:50	Roman Glebov - On bounded degree spanning trees in the random graph
3:55 - 4:15	Maya Stein - Partitioning and covering with monochromatic components
4:20 - 4:40	Tereza Klimosova - Forcibility of graphons with infinitely dimensional space of typical points
4:45 - 5:15	Coffee Break
5:15 - 5:35	Johannes Carmesin - Rota's conjecture for infinite matroids
5:40 - 6:00	Tony Huynh - Hilbert bases of cuts

Friday, December 20

9:30 - 9:50	Frederic Mazoit - The redundant vertex theorem on surfaces
9:55 - 10:15	Yusuke Kobayashi - The generalized terminal backup problem
10:20 - 10:50	Coffee Break
10:50 - 11:10	Ross Kang - List coloring with a bounded palette
11:15 - 11:35	Luke Postle - 3 coloring and 3-list coloring graphs on surface
11:40 - 12:00	Petru Valicov - Variants of edge-coloring
12:00 -	Lunch and departure

Rota's Conjecture for infinite matroids

Johannes Carmesin, University of Hamburg

We use Psi-matroids, which come from topological infinite graph theory, to construct an infinite antichain of infinite binary matroids. These matroids are not finitary.

Polynomial bounds for the Grid-Minor Theorem

Julia Chuzhoy, Toyota Technological Institute

One of the key results in Robertson and Seymour's seminal work on graph minors is the Grid-Minor Theorem (also known as the Excluded Grid Theorem). The theorem states that any graph of treewidth at least k contains a grid minor of size $f(k)$ for some function f . This theorem has found many applications in graph theory and algorithms. The best current quantitative bound, due to recent work of Kawarabayashi and Kobayashi, and Leaf and Seymour, shows that $f(k) = \Omega(\sqrt{\log k / \log \log k})$, while the best known upper bound implies that $f(k) = O(\sqrt{k / \log k})$. In this talk we present the first polynomial relationship between treewidth and grid-minor size by showing that $f(k) = \Omega(k^\delta)$ for some fixed constant $\delta > 0$, and also describe an efficient algorithm to construct such a minor.

Joint work with Chandra Chekuri.

Tangles, brambles, blocks and profiles

Reinhard Diestel, University of Hamburg

We compare various notions of dense parts in sparse graphs to each other - such as tangles, brambles etc - and look for a comprehensive duality theorem that equates their existence to the non-existence of an appropriate tree-decomposition of the graph considered.

Coloring planar graphs with three colors and no large monochromatic components

Louis Esperet, G-SCOP

We prove the existence of a function c such that the vertices of every planar graph with maximum degree Δ can be 3-colored in such a way that each monochromatic component has at most $c(\Delta)$ vertices. This is best possible (the number of colors cannot be reduced and the dependence on the maximum degree cannot be avoided) and answers a question raised by Kleinberg, Motwani, Raghavan, and Venkatasubramanian in 1997. Our result extends to graphs of bounded genus.

This is joint work with Gwenaël Joret.

On bounded degree spanning trees in the random graph

Roman Glebov, ETH

The appearance of certain spanning subgraphs in the random graph is a well-studied phenomenon in probabilistic graph theory. In this talk, we present results on the threshold for the appearance of bounded-degree spanning trees in $G(n, p)$ as well as for the corresponding universality statements. In particular, we show hitting time thresholds for some classes of bounded degree spanning trees. Joint work with Daniel Johannsen and Michael Krivelevich.

Hilbert bases of cuts

Tony Huynh, University of Rome

A Hilbert basis is a set of vectors X such that the integer cone generated by X is the intersection of the lattice generated by X with the cone generated by X .

In this talk, we investigate the set of graphs whose set of cuts is a Hilbert basis. We say that these graphs have the Hilbert cut property. Somewhat surprisingly, we show that the Hilbert cut property is not closed under edge deletions, subdivisions, nor 2-sums. We also show that $K_6 - e$ does not have the Hilbert cut property, and that all graphs with $K_6 - e$ as a minor do not have the Hilbert cut property.

On the other hand, we give conditions under which performing 2-sums does preserve the Hilbert cut property. Using these conditions we obtain some positive results for the Hilbert cut property.

This is joint work with Luis Goddyn and Tanmay Deshpande.

Hamilton connectivity and the scattering number of interval graphs

Tomáš Kaiser, University of West Bohemia

We relate Hamilton connectivity of an interval graph G to its scattering number, defined as

$$sc(G) = \max_X (c(G - X) - |X|),$$

where $c(G - X)$ denotes the number of components of $G - X$ and X ranges over all vertex sets with $c(G - X) \geq 2$. It has been shown by Deogun, Kratsch and Steiner that an interval graph is Hamiltonian if and only if its scattering number is at most 0. We show that an interval graph is Hamilton connected if and only if its scattering number is at most -1 , and we extend this to the so-called k -Hamilton-connectedness. Moreover, we give a linear-time algorithm to find the scattering number of an interval graph, improving an existing $O(n^3)$ time algorithm due to Kratsch, Kloks and Müller. A brief overview of known results relating the scattering number to Hamilton cycles and paths in interval graphs will be included.

This is joint work with Hajo Broersma, Jiří Fiala, Petr A. Golovach, Daniël Paulusma and Andrzej Proskurowski.

List colouring with a bounded palette

Ross Kang, University of Utrecht

Král' and Sgall introduced a notion of list colouring where the lists are composed of colours from some predetermined set – we call this the “palette” – that in particular has a fixed size. This notion was already observed to be related to an extremal parameter for Property B or hypergraph 2-colourability. We relate the notion to a possibly new property we dub Property K. In doing so, we see for instance that there is a graph that is k -choosable if the palette may only have $2k - 1$ colours, but is not $2^{\Omega(k)}$ -choosable in general. We also consider other aspects of the problem. Joint work with Marthe Bonamy.

Forcibility of graphons with infinitely dimensional space of typical points

Tereza Klímová, University of Warwick

Graphons are analytic objects associated with convergent sequences of graphs. Problems from extremal combinatorics and theoretical computer science led to a study of graphons determined by finitely many subgraph densities, which are referred to as finitely forcible. We show that there exists a finitely forcible graphon such that the topological space of its typical points has infinite Lebesgue covering dimension, disproving the conjecture by Lovasz and Szegedy. The talk is based on joint work with Roman Glebov and Daniel Kral.

The generalized terminal backup problem

Yusuke Kobayashi, University of Tokyo

We consider the following network design problem, that we call the Generalized Terminal Backup Problem. Given a graph (or a hypergraph) $G_0 = (V, E_0)$, a set of (at least 2) terminals $T \subseteq V$ and a requirement $r(t)$ for every $t \in T$, find a multigraph $G = (V, E)$ such that $\lambda_{G_0+G}(t, T-t) \geq r(t)$ for any $t \in T$. In the minimum cost version the objective is to find G minimizing the total cost $c(E) = \sum_{uv \in E} c(uv)$, given also costs $c(uv) \geq 0$ for every pair $u, v \in V$. In the degree-specified version the question is to decide whether such a G exists, satisfying that the number of edges is a prescribed value $g(v)$ at each node $v \in V$. We solve the Generalized Terminal Backup Problem in the following two cases.

In the first case we start with the minimum cost version for $c \equiv 1$, which helps solving the degree-specified version by a splitting-off theorem. This splitting-off theorem in turn provides the solution for the minimum cost version in the case when c is node-induced, that is $c(uv) = w(u) + w(v)$ for some node weights w on V .

In the second solved case we turn to the general minimum cost version, and we are able to solve it when G_0 is the empty graph. This includes the Terminal Backup Problem solved by Anshelevich and Karagiozova ($r \equiv 1$) and the Maximum-Weight b -matching Problem ($T = V$). The solution depends on an interesting new variant of a theorem of Lovász and Cherkassky, and on the solution of the so-called Simplex Matching problem.

This is joint work with Attila Bernáth.

FO limits of trees

Dan Kral, University of Warwick

Nesetril and Ossona de Mendez introduced a notion of FO convergence of graph sequences. This notion can be viewed as a unified notion of the existing notions of convergence of dense and sparse graphs. In particular, every FO convergent sequence of graphs is convergent in the sense of left convergence of dense graphs, and every FO convergent sequence of sparse graphs is convergent in the Benjamini-Schramm sense.

During the talk, we first introduce graph limit theory and then present results on the existence of limit objects for FO convergent sequences of graphs we have recently obtained.

The talk is based on a joint work with Martin Kupec and Vojtech Tuma.

Clique Stable separation in perfect graphs with no balanced skew partitions

Aurélien Lagoutte, Ecole Normale Supérieure de Lyon

In 1990, Yannakakis [5] studies the Vertex Packing polytope of a graph, and focuses on the case of perfect graphs. This leads him to the following problem where a *cut* is a bipartition of the vertices of the graph: does there exist a polynomial family F of cuts such that, for every clique K and every stable set S of the graph that do not intersect, there exists a cut (U, W) of F that *separates* K and S , meaning $K \subseteq U$ and $S \subseteq W$? Such a family of cuts separating all the cliques and the stable sets is called a *Clique-Stable set separator* (*CS-separator* for short), and he asks for the existence of a polynomial one. Yannakakis showed that it exists for comparability graphs and their complements, chordal graphs and their complements, Lovász proved it [4] for a generalization of series-parallel graphs and we recently proved it [1] for random graphs, H -free graphs if H is a split graph, and graphs with no induced path of length k and its complement.

Observe that the Strong Perfect Graph Theorem [2] encourages us to use decomposition results: every Berge graph is either in some basic class, or has some kind of decomposition. However the *balanced skew-partition* is a notoriously difficult decomposition to handle. Using *trigraphs*, Chudnovsky, Trotignon, Trunck and Vušković [3] proved that forbidding it inductively preserves some structure.

Using this result, we prove that any Berge graph with no balanced skew-partition admits a $\mathcal{O}(n^2)$ CS-separator. We generalize the method by defining the *generalized k -join* between two trigraphs and showing that it still preserves the existence of a polynomial CS-separator.

This is joint work with Nicolas Bousquet, Stéphan Thomassé, and Théophile Trunck.

References

- [1] N. Bousquet, A. Lagoutte, and S. Thomassé. Clique versus Independent Set, 2013. <http://arxiv.org/abs/1301.2474>, preprint.

- [2] M. Chudnovsky, N. Robertson, P. Seymour, and R. Thomas. The strong perfect graph theorem. *Annals of Mathematics*, 164(1):51–229, 2006.
- [3] M. Chudnovsky, N. Trotignon, T. Trunck, and K. Vušković. Coloring perfect graphs with no balanced skew-partitions. submitted.
- [4] L. Lovász. Stable sets and polynomials. *Discrete Mathematics*, 124(1-3):137–153, 1994.
- [5] M. Yannakakis. Expressing combinatorial optimization problems by linear programs. *J. Comput. Syst. Sci.*, 43(3):441–466, 1991.

On the oriented cycle game

Anita Liebenau, University of Warwick

We consider the following orientation game. OMaker and OBreaker take turns in directing edges of the complete graph on n vertices. If the final tournament contains a directed cycle, then OMaker wins. Otherwise, OBreaker wins. Since this game is drastically in favour of OMaker, OBreaker may direct between one and b edges per round. Then the following question arises naturally. What is the largest bias b^* we can give OBreaker such that OMaker wins the oriented cycle game. Bollobás and Szabó showed that b^* is at least linear. They also observed that $n - 2$ is a trivial upper bound, and conjectured that this is tight when OBreaker has to direct exactly b edges in each round. In 2012, Ben-Eliezer, Krivelevich and Sudakov showed that $b^* > n/2 - 3$. We improve the upper bound and show that for $b^* = 5n/6 + 2$ OBreaker has a winning strategy in the monotone version of the game, and for $b^* = cn$ OBreaker has a winning strategy in the strict version of the game, where $c < 1$ is a constant. Joint work with Dennis Clemens.

Well-quasi-ordering graphs by the topological minor relation: Robertson’s conjecture

Chun-hung Liu, Georgia Institute of Technology

Robertson and Seymour proved that graphs are well-quasi-ordered by the minor relation and the weak immersion relation in the prominent Graph Minors series. That is, given infinitely many graphs, one graph contains another as a minor (or a weak immersion, respectively). However, the topological minor relation does not well-quasi-order graphs. An old conjecture of Robertson states that for every integer k , graphs with no topological minor isomorphic to the graph obtained from a path of length k by doubling every edge are well-quasi-ordered by the topological minor relation. We are able to prove this conjecture, and we will sketch the proof in this talk. This work is joint with Robin Thomas.

Wheel- and antiwheel-free graphs

Frédéric Maffray, G-SCOP

A wheel is a graph that consists of a hole plus a vertex that has at least three neighbors in the hole. There are several interesting and well-characterized classes of wheel-free graphs. However, it was proved recently that deciding if a graph is wheel-free graph is an NP-complete problem. I will focus on the class of graphs such that both the graph and its complement do not contain a wheel and show a complete characterization of this class.

The redundant vertex theorem on surfaces

Frédéric Mazoit, LaBRI

Let $s_1, t_1, \dots, s_k, t_k$ be vertices in a graph G embedded on a surface of genus g . A vertex v of G is “redundant if there exist k vertex disjoint paths linking s_i and t_i in G if and only if such paths also exist in $G - v$. In Graph Minors VII, Robertson and Seymour proved sufficient conditions for a vertex to be redundant but their result involves a constant $l(g, k)$ which is non explicit. This is very unfortunate because this constant is used in many results including algorithms for minor testing, k -disjoint path problem and so on. In this talk, I will outline a proof that $l(g, k) \leq c^{g+k}$ for some explicit constant c .

Subdivisions in 4-connected graphs of large tree-width

Irene Muzi, University of Rome

The grid theorem of Robertson and Seymour proves that graphs of sufficiently large treewidth contain a $r \times r$ grid as a minor. The same does not hold true for grid subdivisions, as any graph G of maximum degree 3 constitutes a counterexample regardless of its treewidth. By restricting the problem to 4-connected graphs, we prove that graphs of sufficiently large treewidth contain either a large grid or a graph obtained by adding an apex vertex to a 3-regular graph of large treewidth. Using analogous techniques we prove that nonplanar graphs of sufficiently large treewidth contain K_5 as a subdivision. This problem is connected to a well known conjecture posed independently by Seymour (1975) and Kelmans (1979) that states that every 5-connected nonplanar graph contains K_5 as a subdivision.

This is joint work with Paul Wollan.

An extension to 3-colorable or Eulerian triangulations

Kenta Ozeki, National Institute for Informatics, Kawarabayashi Large Graph Project

In this talk, we concentrate on the following problem; For a given graph G embedded on a surface F , can one add diagonals to all non-triangular faces of G so that the resultant graph is a 3-colorable triangulation of F ? It was known that this problem generally belongs to NP-complete, but there is a polynomial algorithm to solve it when we restrict ourselves to the case where G is a quadrangulation and F is the sphere. Note that every 3-colorable triangulation is Eulerian.

I will discuss about the problem, and give an algorithm to enumerate all possible extensions to 3-colorable, or Eulerian triangulations for a graph G embedded on a surface such that all faces of G are triangular or quadrangular. This is a joint work with Atsuhiko Nakamoto (Yokohama National University) and Kenta Noguchi (Keio University).

Isolating Induced Subgraphs

Irena Penev, Ecole Normale Supérieure de Lyon

A *cutset* of a graph G is a (possibly empty) subset C of $V(G)$ such that $G \setminus C$ is disconnected. If \mathcal{G} is a hereditary class of graphs (i.e. a class closed under isomorphism and induced subgraphs) and k is a positive integer, then we denote by \mathcal{G}^k the closure of \mathcal{G} under the operation of gluing two graphs along induced subgraphs of size at most k . In particular then, \mathcal{G}^k is closed under disjoint unions, and furthermore, every graph in \mathcal{G}^k either belongs to \mathcal{G} or has a cutset of size at most k .

It may seem reasonable to expect that every graph G in \mathcal{G}^k either belongs to \mathcal{G} , or has a cutset C of size at most k such that some component of $G \setminus C$ belongs to \mathcal{G} . Unfortunately, this is false if $k \geq 2$. In this talk, we present the proof of a similar (but slightly weaker) statement: every graph G in \mathcal{G}^k either belongs to \mathcal{G} or has a cutset C of size at most $2k^2 - 1$ such that some component of $G \setminus C$ belongs to \mathcal{G} .

We also discuss the implications of this result for χ -boundedness. A hereditary class \mathcal{G} is said to be χ -*bounded* if there exists a non-decreasing function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that for all $G \in \mathcal{G}$, $\chi(G) \leq f(\omega(G))$. In 1987, a group of authors [1] proved that if \mathcal{G} is χ -bounded, then so is \mathcal{G}^k . This result was recently strengthened in [2] by improving the χ -bounding function for \mathcal{G}^k . We strengthen the result further by obtaining an even better χ -bounding function for \mathcal{G}^k .

Joint work with Stéphan Thomassé and Nicolas Trotignon.

References

- [1] N. Alon, D.J. Kleitman, M. Saks, P. Seymour, and C. Thomassen. Subgraphs of large connectivity and chromatic number in graphs of large chromatic number. *J. Graph Theory*, 11:367–371, 1987.
- [2] M. Chudnovsky, I. Penev, A. Scott, and N. Trotignon. Substitution and χ -boundedness. *Journal of Combinatorial Theory, Series B*, 103(5):567–586, 2013.

Everything you always wanted to know about the parameterized complexity of Subgraph Isomorphism (but were afraid to ask)

Daniel Marx, MTA SZTAKI

Given two graphs H and G , the Subgraph Isomorphism problem asks if H is isomorphic to a subgraph of G . While NP-hard in general, algorithms exist for various parameterized versions of the problem and the way the algorithm can depend on the parameters is highly nontrivial and subtle. However, the literature contains very few negative results ruling out that certain combination of parameters cannot be exploited algorithmically. Our goal is to systematically investigate the possible parameterized algorithms that can exist for Subgraph Isomorphism.

We develop a framework involving 10 relevant parameters for each of H and G (such as treewidth, pathwidth, genus, maximum degree, number of vertices, number of components, etc.), and ask if an algorithm with running time

$$f_1(p_1, p_2, \dots, p_\ell) \cdot n^{f_2(p_{\ell+1}, \dots, p_k)}$$

exists, where each of p_1, \dots, p_k is one of the 10 parameters depending only on H or G . We show that *all* the questions arising in this framework are answered by a set of 11 maximal positive results (algorithms) and a set of 17 maximal negative results (hardness proofs); some of these results already appear in the literature, while others are new in this paper.

On the algorithmic side, our study reveals for example that an unexpected combination of bounded degree, genus, and feedback vertex set number of G gives rise to a highly nontrivial algorithm for Subgraph Isomorphism. On the hardness side, we present W[1]-hardness proofs under extremely restricted conditions, such as when H is a bounded-degree tree of constant pathwidth and G is a planar graph of bounded pathwidth.

(joint work with Michal Pilipczuk)

Network sparsification for Steiner problems on planar and bounded-genus graphs

Marcin Pilipczuk, University of Warsaw

We propose polynomial-time algorithms that sparsify planar and bounded-genus graphs while preserving optimal solutions to Steiner problems. Our main contribution is a polynomial-time algorithm that, given a graph G embedded on a surface of genus g and a designated face f bounded by a simple cycle of length k , uncovers a set of edges F of size polynomial in g and k that contains an optimal Steiner tree for any set of terminals that is a subset of the vertices of f .

We apply this general theorem to prove that: * given a graph G embedded on a surface of genus g and a terminal set S , one can in polynomial time find a set F that contains an optimal Steiner tree T for S and that has size polynomial in g and $|E(T)|$; * an analogous result holds for the Steiner Forest problem; * given a planar graph G and a terminal set S , one can in polynomial time find a set F that contains an optimal edge multiway cut C separating S (i.e., a cutset that intersects any path with endpoints in different terminals from S) and that has size polynomial in $|C|$.

In the language of parameterized complexity, these results imply the first polynomial kernels for Steiner Tree and Steiner Forest on planar and bounded-genus graphs (parameterized by the size of the tree and forest, respectively) and for Edge Multiway Cut on planar graphs (parameterized by the size of the cutset).

Joint work with Michal Pilipczuk, Piotr Sankowski and Erik Jan van Leeuwen.

Minimum Bisection is fixed-parameter tractable

Michal Pilipczuk, University of Bergen

In the classic Minimum Bisection problem we are given as input a graph G and an integer k .

The task is to determine whether there is a partition of $V(G)$ into two parts A and B such that $||A| - |B|| \leq 1$ and there are at most k edges with one endpoint in A and the other in B . We give an algorithm for Minimum Bisection with running time $O(2^{O(k^3)} * n^3 \log^3 n)$, which is the first fixed parameter tractable algorithm for Minimum Bisection. At the core of our algorithm lies a new decomposition theorem that states that every graph G can be decomposed by small separators into parts where each part is “highly connected” in the following sense: any cut of bounded size can separate only a limited number of vertices from each part of the decomposition.

3-Coloring and 3-List-Coloring Graphs on Surface

Luke Postle, Emory University

Thomassen proved that for every surface, there exist only finitely many 4-critical graphs of girth at least five embeddable on that surface. This implies that locally planar graphs are 3-colorable and that there exists a linear-time algorithm for deciding 3-colorability on a fixed surface.

Dvorak, Kral and Thomas proved a stronger version using discharging: that the number of vertices in such graphs is linear in genus. We discuss two new proofs of this linear bound, one using list-coloring and one using the potential technique.

An edge variant of the Erdős - Pósa property

Jean-Florent Raymond, University of Warsaw

A variant of the Erdős–Pósa Theorem [1] states that there is a function $f: \mathbb{N} \rightarrow \mathbb{N}$ such that for every $k \in \mathbb{N}$ and for every graph G , either G contains k edge-disjoint cycles, or there is a set of $f(k)$ edges in G meeting all cycles of G .

For every $r \in \mathbb{N}$, we denote by θ_r the multigraph with two vertices and r parallel edges. These graphs can be seen as a generalization of cycles. A minor model of a graph H in a graph G is a subgraph of G that can be contracted to H . Our main result is the following.

Theorem 1 *For every $r \in \mathbb{N}^*$, there is a function $f_{\theta_r}: \mathbb{N} \rightarrow \mathbb{N}$ such that for every $k \in \mathbb{N}^*$, every graph G has k edge-disjoint minor models of θ_r , or a set of $f_{\theta_r}(k)$ edges hitting all the minor models of θ_r it contains. Furthermore, $f_{\theta_r}(k) = O(k^3 r^3)$.*

The proof contains two main ingredients. The first is a reduction of the problem to biconnected graphs and the second is a suitable extension of the techniques in [3] for edge packings and coverings. We also use a result of [2].

As it has been done in the past with the classic version or the Erdős–Pósa Theorem, we tried to extend its edge variant to other graph classes than cycles, namely minor models of graphs θ_r . Two questions still remain unanswered:

1. can we find a better estimation of the function f_{θ_r} ?
2. is it possible to characterize graphs satisfying this edge version?

This is joint work with Ignasi Sau and Dimitrios M. Thilikos.

References

- [1] Paul Erdős and Louis Pósa. On independent circuits contained in a graph. *Canad. J. Math.*, 17:347–352, 1965.
- [2] Fedor V. Fomin, Daniel Lokshtanov, Neeldhara Misra, Geevarghese Philip, and Saket Saurabh. Quadratic upper bounds on the Erdős–Pósa property for a generalization of packing and covering cycles. *Journal of Graph Theory*, 74:417–424, 2013.
- [3] Fedor V. Fomin, Saket Saurabh, and Dimitrios M. Thilikos. Strengthening Erdős–Pósa property for minor-closed graph classes. *Journal of Graph Theory*, 66(3):235–240, 2011.

How many edges force an H minor?

Bruce Reed, McGill University

We discuss $d(H)$, the infimum of those d such that every graph G of average degree d has H as a minor. Motivated by Hadwiger’s conjecture, Mader proved that $d(H)$ is finite for every d . Precise bounds have been obtained on $d(H)$ for dense H by Thomason and Myers. We focus on sparse H .

Disjoint Dijoins

Paul Seymour, Princeton University

A “dijoin” in a digraph is a set of edges meeting all directed cuts. The Lucchesi-Younger theorem says that if every dijoin has size at least k , then there are k pairwise disjoint directed cuts, and in 1976 Woodall proposed a dual statement, that if every directed cut has size at least k then there are k pairwise disjoint dijoins. This is still open. But Schrijver showed that it becomes false if we give the edges nonnegative integer capacities; it is not always true that if every directed cut has total capacity at least k , then there are k dijoins using each edge at most its capacity.

In Schrijver’s example, and in all others known, the edges with positive capacity form a disconnected graph, and perhaps the capacitated version is true if they form a connected graph. We have several partial results on this. Joint work with Maria Chudnovsky, Katherine Edwards, Ringi Kim and Alex Scott.

Colouring higher dimensional projective quadrangulations

Matěj Stehlík, University Joseph Fourier

A remarkable result of Youngs from 1996 asserts that every quadrangulation of the projective plane is either bipartite or 4-chromatic. We extend the definition of quadrangulation to higher dimensions, and generalise Young’s result by proving that any quadrangulation of the projective n -space is at least $(n+2)$ -chromatic, unless it is bipartite. Higher dimensional projective quadrangulations form a rich family of graphs, which includes all complete graphs and generalised Mycielski graphs. We give a new proof of the Lovász-Kneser theorem by showing that certain

projective quadrangulations are homomorphic to the Schrijver graphs. We also show that unlike in the 2-dimensional case, the chromatic number of n -dimensional projective quadrangulations cannot be bounded by any function of n . This talk is based on joint work with Tomáš Kaiser.

Partitioning and covering with monochromatic components

Maya Stein, University of Chile

Consider a complete graph K_n whose edges are coloured with r colours (this need not be a proper edge-colouring). I will speak about several conjectures/results on how we can cover or partition the vertex set of K_n with connected monochromatic subgraphs. In particular, how many of these subgraphs are needed, and can we require them to take some special form (cycles or paths, for instance). Part of this is joint work with D. Conlon.

To be announced

Blair Sullivan, North Carolina State University

Cliques and stable sets in proper graph classes

Stéphan Thomassé, Ecole Normale Supérieure de Lyon

The aim of this talk is to present two results related to the structure of cliques and stable sets in H -free graphs. The key-point here is that we do not rely on an explicit structure or decomposition of H -free graphs to reach our goal.

The existence of extended formulations for the stable set polytope of perfect graphs would imply the existence of a polynomial clique stable separation, i.e. a family of n^c vertex bipartitions separating cliques from stable sets in any perfect graph of size n , where c is some constant. In other words, for each pair of disjoint clique K and stable set S , one of the bipartition of the family has K in one side, and S in the other side. It could be true that every H -free class has the polynomial clique stable separation. We prove it in the case when H is any split graph.

The Erdős-Hajnal conjecture asserts for every H , there exists a constant c such that any H -free graphs have either a clique or a stable set of size n^c . As the previous question, this conjecture is wide open for $H = C_5$. I will provide here a proof when forbidding P_k and its complement as induced subgraphs.

Joint work with Marthe Bonamy, Nicolas Bousquet and Aurélie Lagoutte.

The stable set problem is FPT in bull-free graphs

Nicolas Trotignon, CNRS Ecole Normale Supérieure de Lyon

The bull is the graph obtained from the triangle by adding two pendant non-adjacent edges. A graph is bull-free if it does not contain a bull as an induced subgraph. Finding a maximum stable set in a bull-free graph is a NP-hard problem. We prove that this problem is FPT. Our proof relies on the decomposition theorem for bull-free graph due to Maria Chudnovsky. This is a joint work with Stéphan Thomassé and Krisitina Vušković.

Variants of edge-colouring

Petru Valicov, ENS de Lyon

A strong edge-colouring of a graph G is a partitioning of the edge set of G into induced matchings. We first present an overview of the results on strong edge-colouring for different graph classes. Then we introduce the notion of k -intersection edge-colouring which captures both the notion of edge-colouring and the strong edge-colouring. To be precise, k -intersection edge-colouring of a graph is a proper edge-colouring such that for every pair of adjacent vertices, u and v , the set of colours on edges incident to u has at most k colours in common with the set of colours on edges incident to v . Thus $\Delta(G)$ -intersection edge-colouring is just a proper edge-colouring and 1-intersection edge-colouring is the strong edge-colouring.

As usual, we are interested in the minimum number of colours needed for k -intersection edge-colouring of a graph. It turns out that the problem is difficult even for some basic families of graphs like the complete graphs and is related to some interesting problems in combinatorial set theory. We present few results in this direction and we show some complexity result for the general problem.

These include joint results with different sets of authors from France, India, Japan and Taiwan.

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