# 2016 Southern Italian Workshop on Algorithms and Graphs

September 25-30

*Schedule and Abstracts*

**Monday**

9:25 - 9:30 Announcements

9:30 - 10:30 Norin *Improperly Coloring $K_t$ Minor-Free Graphs*

10:30 - 11:00 *coffee*

11:00 - 11:20 Aboulker *Subdivisions in digraphs of large out-degree or large dichromatic number*

11:25 - 11:45 Kim *Unavoidable subtournaments in tournaments with large chromatic number*

11:50 - 12:10 Kwon *Coloring graphs without fan vertex-minors and graphs without cycle pivot-minors*

12:10 - 4:00 *Break*

4:00 - 4:20 Oum *Defective coloring of graphs excluding either a complete bipartite graph or the* 1*-subdivision of a complete graph*

4:25 - 4:45 Le *Forcing clique immersions through chromatic number*

4:50 - 5:10 Mazzuoccolo *On cubic graphs admitting a bisection with small monochromatic components*

5:10 - 5:40 *coffee*

5:40 - 6:00 Bonamy *Kempe equivalence of colourings*

6:05 - 6:25 Ozeki *Kempe equivalence of 3-edge-colorings in cubic graphs on the projective plane*

**Tuesday**

9:25 - 9:30 Announcements

9:30 - 10:30 Scott *Induced subgraphs of graphs with large chromatic number*

10:30 - 11:00 *coffee*

11:00 - 12:00 Seymour *Banana trees and $\chi$-boundedness*

12:00 - 4:00 *Break*

4:00 - 4:20 Penev *Stable sets in $\{ISK4, wheel\}$-free graphs*

4:25 - 4:45 Radovanovic *On graphs that do not contain a theta nor a wheel*

4:50 - 5:10 Spirkl *Approximately coloring graphs without long induced paths*

5:10 - 5:40 *coffee*

5:40 - 6:00 Boncompagni *A generalization of line graphs of triangle-free graphs and chordal graphs*

6:05 - 6:25 Vuskovic *Algorithms for (cap, even-hole)-free graphs*

6:30 - 6:50 Zhong *Three-coloring and list three-coloring of graphs without induced paths on seven vertices*

**Wednesday**

9:25 - 9:30 Announcements

9:30 - 10:30 Adler - *PAC learning of definable sets in graph classes*

10:30 - 11:00 *coffee*

11:00 - 11:20 Michal Pilipczuk *Guided treewidth and Courcelle's conjecture*

11:25 - 11:45 Thomasse *Sparse VC-dimension*

11:50 - 12:10 Postle *Bounded Diameter Arboricity*

12:10 - 4:30 *Break*

4:30 Departure for excursion to Cisternino

**Thursday**

9:25 - 9:30 Announcements

9:30 - 10:30 Schaudt *News on the list k-colorability problem in $H$-free graphs*

10:30 - 11:00 *coffee*

11:00 - 11:20 Stein *Tree embeddings with minimal degree conditions*

11:25 - 11:45 Pfender *Semidefinite Programming and Ramsey Numbers*

11:50 - 12:10 Bowler *The reconstruction conjecture for infinite graphs*

12:10 - 4:00 *Break*

4:00 - 4:20 Joos *The Tree Packing Conjecture*

4:25 - 4:45 Klimosova *Edge-partitioning graphs into paths and trees*

4:50 - 5:10 Cames van Batenburg *Packing graphs of bounded codegree*

5:10 - 5:40 *coffee*

5:40 - 6:00 Edwards *Approximating p-centres in δ-hyperbolic graphs*

6:05 - 6:25 Fiorini *Improved Approximation Algorithms for Hitting 3-Vertex Paths*

6:30 - 6:50 Marcin Pilipczuk *Subexponential parameterized algorithms for planar and apex-minor-free graphs via low treewidth pattern covering*

**Friday**

9:25 - 9:30 Announcements

9:30 - 10:30 Lokshtanov *A Linear Time Parameterized Algorithm for Directed Feedback Vertex Set*

10:30 - 11:00 *coffee*

11:00 - 11:20 Chekuri *Routing Problems with Symmetric Demands and Directed Treewidth*

11:25 - 11:45 Kobayashi *The directed disjoint shortest paths problem*

11:50 - 12:10 Marx *The Complexity Landscape of Fixed-Parameter Directed Steiner Network Problems*

12:10 Lunch and departure

# Subdivisions in digraphs of large out-degree or large dichromatic number

Pierre Aboulker

**Abstract**

A classic result of Mader states that there exists an integer $g(k)$ such that every graph with average degree at least $g(k)$ contains a subdivision of $K_k$. In particular, it implies that if the minimum degree, or the chromatic number, or the connectivity of a graph is at least $g(k)$, then it contains a subdivision of $K_k$.

We will discuss analogues of these results in the world of digraphs where the situation is less idyllic. We will insist on two parameters: the dichromatic number and the minimum outdegree.

The *dichromatic number* of a digraph $D$ is the smallest integer $k$ such that there exists a $k$-partition $(V_1, \dots, V_k)$ of $V(D)$ such that $D[V_i]$ is acyclic for $1 \leq i \leq k$. We prove that there exists an integer $f_1(k)$ such that every digraph with dichromatic number at least $f_1(k)$ contains a subdivision of the complete digraph on $k$ vertices.

We will also prove that there exists an integer $f_2(k)$ such that every digraph with minimum outdegree $f_2(k)$ contains a subdivision of every in-arborescence on $k$ vertices. This is a step toward a conjecture of Mader that states that there exists an integer $f_3(k)$ such that every digraph with minimum outdegree $f_3(k)$ contains a subdivision of the transitive tournament on $k$ vertices.

This is a joint a work with Nathann Cohen, Frédéric Havet, William Lochet, Phablo F. S. Moura and Stéphan Thomassé.

# PAC learning of definable sets in graph classes

Isolde Adler

**Abstract**

In machine learning, the problem of "concept learning" is to identify an unknown set from a given concept class (i.e. collection of sets) algorithmically. In the model of "probably approximately correct" (PAC) learning, the learner receives a number of samples and must be able to identify the unknown set approximately, in a probabilistic sense. It is well-known the the sample size for PAC learning is characterised by the Vapnik-Cervonenkis (VC) dimension of the concept class. We are interested in the VC dimension of concept classes that are definable in some logic on classes of graphs. In 2004, Grohe and Turán showed that for any subgraph closed class $C$, monadic second-order definable concept classes have bounded VC dimension on $C$ if and only if $C$ has bounded tree-width. We show that for any subgraph closed class $C$, first-order definable concept classes have bounded VC dimension on $C$ if and only if $C$ is nowhere dense. This is joint work with Hans Adler.

# Kempe equivalence of colourings

Marthe Bonamy

**Abstract**

Given a colouring of a graph, a Kempe change is the operation of picking a maximal bichromatic subgraph and switching the two colours in it. Two colourings are Kempe equivalent if they can be obtained from each other through a series of Kempe changes. Kempe changes were first introduced in a failed attempt to prove the Four Colour Theorem, but they proved to be a powerful tool for other colouring problems. They are also relevant for more applied questions, most notably in theoretical physics. Consider a graph with no vertex of degree more

than some integer $D$. In 2007, Mohar conjectured that all its $D$-colourings are Kempe-equivalent. Since 1981, we know from Las Vergnas and Meyniel that this is true if the graph is not $D$-regular. Feghali, Johnson and Paulusma proved in 2015 that 3-regular graphs also satisfy the conjecture, with the exception of the 3-prism (two triangles joined by a matching) which disproves it. We settle the remaining cases by proving that all $k$-colourings of a $k$-regular graph are Kempe equivalent for $k$ at least 4. This is a joint work with Nicolas Bousquet (CNRS, G-SCOP, France), Carl Feghali (IRIF, France) and Matthew Johnson (Durham University, UK).

# A generalization of line graphs of triangle-free graphs and chordal graphs

Valerio Boncompagni

**Abstract**

In this talk we describe the structure of a hereditary graph class that generalizes line graphs of triangle-free graphs as well as chordal graphs. This is a class of graph defined by forbidding (some of the so-called) Truemper configurations as induced subgraphs. As a consequence of the decomposition theorem we present, we prove that every $K_4$-free graph in our class is 4-colorable. Also, we give a polynomial time recognition algorithm for the class.

This is a joint work with M. Radovanovic and K. Vuskovic.

# The reconstruction conjecture for infinite graphs

Nathan Bowler

**Abstract**

An important open question in the theory of finite graphs is whether it is possible to reconstruct any large enough finite graph from the family of subgraphs which can be obtained by removing individual vertices. The same problem for various classes of infinite graphs, such as trees or locally finite connected infinite graphs, has also remained open for the last few decades. We resolve these questions about infinite graphs by exhibiting a locally finite tree which is not reconstructible.

# Packing graphs of bounded codegree

Wouter Cames van Batenburg

**Abstract**

Two graphs $G_1$ and $G_2$ of order $n$ are said to *pack* if there exist injective mappings of their vertex sets into $[n]$ such that the images of their edge sets are disjoint. A longstanding conjecture due to Bollobás and Eldridge and, independently, Catlin, asserts that, if $(\Delta(G_1)+1)(\Delta(G_2)+1) \leq n+1$, then $G_1$ and $G_2$ pack. We consider the validity of this assertion under the additional assumption that $G_1$ or $G_2$ has bounded codegree. In particular, we prove for all $t \geq 2$ that if $G_1$ does not contain a copy of the complete bipartite graph $K_{2,t}$ and $\Delta_1 > 17t \cdot \Delta_2$, then $(\Delta(G_1)+1)(\Delta(G_2)+1) \leq n+1$ implies that $G_1$ and $G_2$ pack.

We also prove that the conjecture holds under the additional assumptions that both graphs have large enough maximum degree and do not contain 4-, 6- and 8-cycles.

This is joint work with Ross Kang.

# Routing Problems with Symmetric Demands and Directed Treewidth

Chandra Chekuri

**Abstract**

There has been a very fruitful exchange of ideas between work on approximation algorithms for routing problems and graph structure theory, specifically in the context of treewidth. We will describe initial work and progress on extending some of these connections to the case of directed graphs. The routing problem of interest here involves symmetric demands.

Joint work with Alina Ene and Marcin Pilipczuk

# Approximating $p$-centres in $\delta$-hyperbolic graphs

Katherine Edwards

**Abstract**

We describe a graph clustering algorithm that is practical for large $\delta$-hyperbolic graphs. The $p$-centre problem is at the heart of many graph clustering algorithms, but it is NP-hard in general. It has recently been shown that many graphs arising from real-world networks have very small hyperbolic constants so it is interesting to consider this problem on such graphs. We provide a quasilinear time algorithm for $p$-centers with an additive error at most 3 times the input graph's hyperbolic constant. Specifically, for the graph $G = (V, E)$ with $n$ vertices, $m$ edges and hyperbolic constant $\delta$, we construct an algorithm for $p$-centers in time $O(p(\delta + 1)(n + m)\log(n))$ with radius not exceeding $r_p + \delta$ when $p \leq 2$ and $r_p + 3\delta$ when $p \geq 3$, where $r_p$ are the optimal radii. Prior work identified $p$-centers with accuracy $r_p + \delta$ but with time complexity $O((n^3 \log n + n^2 m)\log(diam(G)))$ which is impractical for large graphs.

# Improved Approximation Algorithms for Hitting 3-Vertex Paths

Samuel Fiorini

**Abstract**

We study the problem of deleting a minimum cost set of vertices from a given vertex-weighted graph in such a way that the resulting graph has no induced path on three vertices. This problem is often called cluster vertex deletion in the literature and admits a straightforward 3-approximation algorithm since it is a special case of the vertex cover problem on a 3-uniform hypergraph. Very recently, You, Wang, and Cao (2015) described an efficient 5/2-approximation algorithm for the unweighted version of the problem. Our main result is a 7/3-approximation algorithm for arbitrary weights, using the local ratio technique. We further conjecture that the problem admits a 2-approximation algorithm and give some support for the conjecture. This is in sharp constrast with the fact that the similar problem of deleting vertices to eliminate all triangles in a graph is known to be UGC-hard to approximate to within a ratio better than 3, as proved by Guruswami and Lee (2014)

This is joint work with Gwenaël Joret (Brussels) and Oliver Schaudt (Cologne).

# The Tree Packing Conjecture

Felix Joos

**Abstract**

The famous tree packing conjecture of Gyárfás and Lehel states that for every collection $T_1, \dots, T_n$ of trees such that $|T_i| = i$, there is a decomposition of the edge set of $K_n$ into $T_1, \dots, T_n$. I survey some recent results on this conjecture including the solution of this conjecture for bounded degree trees.

This is joint work with Kim, Kühn, and Osthus.

# Unavoidable subtournaments in tournaments with large chromatic number

Ringi Kim

**Abstract**

For a tournament $T$, the *chromatic number* of $T$ is the minimum number of transitive sets with union $V(T)$. We say a set $\mathcal{H}$ of tournaments is *heroic* if there exists $c$ such that every tournament excluding all members of $\mathcal{H}$ has chromatic number at most $c$. Berger et al. explicitly characterized all heroic sets of size one. In this talk, we study heroic sets of size two.

# Edge-partitioning graphs into paths and trees

Tereza Klimošová

**Abstract**

In 2006, Barat and Thomassen conjectured that for a fixed tree $T$, every sufficiently edge-connected graph with the number of edges divisible by $|E(T)|$ has a $T$-decomposition. That is, the edge set of the graph can be partitioned into isomorphic copies of $T$. The conjecture was recently proven by Bensmail, Harutyunyan, Le, Merker and Thomassé.

Bensmail, Harutyunyan, Le, and Thomassé posed a strengthened version of the conjecture of Barat and Thomassen, that for a fixed tree $T$, every graph with sufficiently high degree and with the number of edges divisible by $|E(T)|$ has a $T$-decomposition if it is sufficiently highly edge-connected in terms of the maximal degree of $T$. They proved the strengthened conjecture for $T$ being a path.

The talk will contain several extensions of the results above. We give the optimum edge-connectivity bound of the strengthened version of the Barat-Thomassen conjecture for paths and we disprove the conjecture for trees of maximal degree at least three. We also prove a relaxed version of the conjecture, showing that for two fixed trees $T$ and $T'$ with coprime numbers of edges, every connected graph with sufficiently high degree has a $\{T, T'\}$-decomposition.

# The directed disjoint shortest paths problem

Yusuke Kobayashi

**Abstract**

In the $k$ disjoint shortest paths problem, we are given a graph and its vertex pairs $(s_1, t_1), \ldots, (s_k, t_k)$, and the objective is to find $k$ pairwise disjoint paths $P_1, \ldots, P_k$ such that each path $P_i$ is a shortest path from $s_i$ to $t_i$, if they exist. If the length of each edge is equal to zero, then this problem amounts to the disjoint paths problem, which is one of the well-studied problems in algorithmic graph theory and combinatorial optimization. Eilam-Tzoreff (1998) focused on the case when the length of each edge is positive, and showed that the undirected version of the 2 disjoint shortest paths problem can be solved in polynomial time. Polynomial solvability of the directed version was posed as an open problem in the same paper. In this talk, we solve this problem affirmatively, that is, we give a first polynomial time algorithm for the directed version of the 2 Disjoint Shortest Paths Problem when the length of each edge is positive. This is joint work with Kristóf Bérczi.

# Coloring graphs without fan vertex-minors and graphs without cycle pivot-minors

O-joung Kwon

**Abstract**

Vertex-minor and pivot-minor relations are graph containment relations developed on the structural theory on rank-width. So far, the understanding of $H$-vertex-minor free graphs or $H$-pivot-minor free graphs are very restricted; for instance, we do not know whether $H$-verter-minor free graphs have bounded rank-width even for a path $H$. Other than bounding rank-width, there are also interesting questions related with these operations. For instance, Geelen conjectured that for fixed graph $H$, the class of $H$-vertex-minor free graphs is $\chi$-bounded.

A fan $F_k$ is a graph that consists of an induced path on $k$ vertices and an additional vertex that is adjacent to all vertices of the path. We prove that for fixed $k$, the class of $F_k$-vertex-minor free graphs is $\chi$-bounded. We also prove that for fixed $k \geq 3$, the class of graphs having no pivot-minor isomorphic to a cycle of length $k$ is $\chi$-bounded.

This is joint work with Ilkyoo Choi and Sang-il Oum.

# Forcing clique immersions through chromatic number

Tien-Nam Le

**Abstract**

Building on recent work of Dvořák and Yepremyan, we show that every simple graph of minimum degree $7t + 7$ contains $K_t$ as an immersion and that every graph with chromatic number at least $3.54t + 4$ contains $K_t$ as an immersion.

This is joint work with Paul Wollan.

# A Linear Time Parameterized Algorithm for Directed Feedback Vertex Set

Daniel Lokshtanov

**Abstract**

In the Directed Feedback Vertex Set (DFVS) problem, the input is a directed graph $D$ on $n$ vertices and $m$ edges, and an integer $k$. The objective is to determine whether there exists a set of at most $k$ vertices intersecting every directed cycle of $D$. Whether or not DFVS admits a fixed parameter tractable (FPT) algorithm was considered the most important open problem in parameterized complexity until Chen, Liu, Lu, O'Sullivan and Razgon [JACM 2008] answered the question in the affirmative. They gave an algorithm for the problem with running time $O(k! * 4^k * k^4 nm)$. Since then, no faster algorithm for the problem has been found. In this talk I will outline the first linear time algorithm for DFVS. Our algorithm has running time $O(k! * 4^k * k^5 * (n + m))$.

Based on joint work with M.S. Ramanujan and S. Saurabh

# The Complexity Landscape of Fixed-Parameter Directed Steiner Network Problems

Dániel Marx

**Abstract**

Given a directed graph $G$ and a list $(s_1, t_1)$, ..., $(s_k, t_k)$ of terminal pairs, the Directed Steiner Network problem asks for a minimum-cost subgraph of $G$ that contains a directed $s_i \to t_i$ path for every $1 \leq i \leq k$. The special case Directed Steiner Tree (when we ask for paths from a root $r$ to terminals $t_1$, ..., $t_k$) is known to be fixed-parameter tractable parameterized by the number of terminals, while the special case Strongly Connected Steiner Subgraph (when we ask for a path from every $t_i$ to every other $t_j$) is known to be W[1]-hard parameterized by the number of terminals. We systematically explore the complexity landscape of directed Steiner problems to fully understand which other special cases are FPT or W[1]-hard. Formally, if $\mathcal{H}$ is a class of directed graphs, then we look at the special case of Directed Steiner Network where the list $(s_1, t_1)$, ..., $(s_k, t_k)$ of requests form a directed graph that is a member of $\mathcal{H}$. Our main result is a complete characterization of the classes $\mathcal{H}$ resulting in fixed-parameter tractable special cases: we show that if every pattern in $\mathcal{H}$ has the combinatorial property of being "transitively equivalent to a bounded-length caterpillar with a bounded number of extra edges," then the problem is FPT, and it is W[1]-hard for *every* recursively enumerable $\mathcal{H}$ not having this property. This complete dichotomy unifies and generalizes the known results showing that Directed Steiner Tree is FPT [Dreyfus and Wagner, *Networks* 1971], Strongly Connected Steiner Subgraph is W[1]-hard [Guo et al., *SIAM J. Discrete Math.* 2011], and Directed Steiner Network is solvable in polynomial-time for constant number of terminals [Feldman and Ruhl, *SIAM J. Comput.* 2006], and moreover reveals a large continent of tractable cases that were not known before.

This is joint work with Andreas Emil Feldmann.

# On cubic graphs admitting a bisection with small monochromatic components

Giuseppe Mazzuoccolo

**Abstract**

Circular nowhere-zero $r$-flows in a cubic graph $G = (V, E)$ and the existence of certain bisections (partitions into two subsets of the same cardinality) of the vertex set $V$ are strictly related. In particular, a circular nowhere-zero $r$-flow in $G$ implies a bisection, where every connected subgraph on $r - 1$ vertices intersects both parts of the bisection. This is related to a recent conjecture of Ban and Linial, stating that any bridgeless cubic graph, other than the Petersen graph, admits a bisection, where the graph induced by each part of the bisection consists of connected components on at most two vertices. Here, we present some recent progress on Ban and Linial conjecture.

# Improperly Coloring $K_t$ Minor-Free Graphs

Sergey Norin

**Abstract**

We show that for every $t > 0$ there exists a constant $c = c(t)$ such that, if a graph G does not contain $K_t$ as a minor, then its vertex set can be partitioned into at most $t - 1$ parts such that every part induces a subgraph with maximum component of size at most $c$. This relaxation of Hadwiger's conjecture improves previous results of Kawarabayashi and Mohar, Wood, and Liu and Oum, who proved that the same conclusion holds for partitions into $31t/2$, $7t/2$ and $3t$ parts respectively. We also discuss applications of our results to extremal questions on bootstrap percolation for minor-closed graph families.

Based on joint work with Zdenek Dvorak.

# Defective coloring of graphs excluding either a complete bipartite graph or the 1-subdivision of a complete graph

Sang-il Oum

**Abstract**

In 1987, Archdeacon proved that graphs embeddable on a fixed surface can be colored by 3 colors such that each color class induces a subgraph of bounded maximum degree. In 2015, Edwards, Kang, Kim, Oum, and Seymour proved that graphs with no $K_{t+1}$-minor can be colored by $t$ colors so that each color class induces a subgraph of bounded maximum degree.

We prove a common generalization of these theorems, which implies previously unknown results such as the following: graphs with no $K_t$-topological minor and no cycle of length 4 can be colored with 2 colors so that each color class has bounded maximum degree.

This is a joint work with Patrice Ossona de Mendez and David R. Wood.

# Kempe equivalence of 3-edge-colorings in cubic graphs on the projective plane

Kenta Ozeki

**Abstract**

Let $G$ be a cubic graph with 3-edge-coloring. For an alternating cycle (i.e. a 2-edge-colored cycle) $D$, a *Kempe switch (at $D$)* is an operation to obtain another 3-edge-coloring by changing the colors of $E(D)$. Two 3-edge-colorings are *Kempe equivalent* if one is obtained from the other by a sequence of Kempe switches. If $G$ is embedded on the plane or the projective-plane, Kempe equivalence is related to topological properties of them. We show that bipartite cubic graphs $G$ on the projective-plane admits only one Kempe equivalent class if and only if the dual $G^*$ is not vertex-4-colorable.

# Stable sets in {ISK4,wheel}-free graphs

Irena Penev

**Abstract**

An *ISK4* is an induced subdivision of the complete graph on four vertices. A *wheel* is a cycle together with a vertex that has at least three neighbors in the cycle. An {*ISK4,wheel*}*-free* graph is a graph that contains no ISK4 and no wheel as an induced subgraph. We present a polynomial-time algorithm that finds the maximum weight of a stable set in a weighted {ISK4,wheel}-free graph (with non-negative integer weights). The algorithm relies on a decomposition theorem for {ISK4,wheel}-free "trigraphs."

Joint work with Martin Milanič and Nicolas Trotignon.

# Semidefinite Programming and Ramsey Numbers

Florian Pfender

**Abstract**

We use the theory of flag algebras to find new upper bounds for small Ramsey numbers. This is joint work with Bernard Lidický.

# Subexponential parameterized algorithms for planar and apex-minor-free graphs via low treewidth pattern covering

Marcin Pilipczuk

**Abstract**

We prove the following theorem. Given a planar graph $G$ and an integer $k$, it is possible in polynomial time to randomly sample a subset $A$ of vertices of $G$ with the following properties:

- $A$ induces a subgraph of $G$ of treewidth $O(\sqrt{k}\log k)$, and
- for every connected subgraph $H$ of $G$ on at most $k$ vertices, the probability that $A$ covers the whole vertex set of $H$ is at least $(2^{O(\sqrt{k}\log^2 k)} \cdot n^{O(1)})^{-1}$, where $n$ is the number of vertices of $G$.

Together with standard dynamic programming techniques for graphs of bounded treewidth, this result gives a versatile technique for obtaining (randomized) subexponential parameterized algorithms for problems on planar graphs, usually with running time bound $2^{O(\sqrt{k}\log^2 k)}n^{O(1)}$. The technique can be applied to problems expressible as searching for a small, connected pattern with a prescribed property in a large host graph; examples of such problems include Directed $k$-Path, Weighted $k$-Path, Vertex Cover Local Search, and Subgraph Isomorphism, among others. Up to this point, it was open whether these problems can be solved in subexponential parameterized time on planar graphs, because they are not amenable to the classic technique of bidimensionality. Furthermore, all our results hold in fact on any class of graphs that exclude a fixed apex graph as a minor, in particular on graphs embeddable in any fixed surface.

Joint work with Fedor Fomin, Daniel Lokshtanov, Dániel Marx, Michał Pilipczuk and Saket Saurabh.

## Guided treewidth and Courcelle's conjecture

Michał Pilipczuk

**Abstract**

During the talk we will present the combinatorial tools used in the recent resolution of Courcelle's conjecture, which states that graph properties recognizable by automata working on tree decompositions are CMSO-definable on graphs of bounded treewidth. The main focus will be on the new notions of guided treewidth and tree decompositions captured by guidance systems, and their relation to standard treewidth and pathwidth. In particular, we will sketch the proof that guided treewidth is bounded by a function of the pathwidth of a graph, which is the cornerstone of the proof of Courcelle's conjecture.

The talk will be based on joint work with Mikołaj Bojańczyk.

## Bounded Diameter Arboricity

Luke Postle

**Abstract**

We introduce the notion of *bounded diameter arboricity*. Specifically, the *diameter-d arboricity* of a graph is the minimum number $k$ such that the edges of the graph can be partitioned into $k$ forests each of whose components has diameter at most $d$. A class of graphs has bounded diameter arboricity $k$ if there exists a natural number $d$ such that every graph in the class has diameter-$d$ arboricity at most $k$. We conjecture that the class of graphs with arboricity at most $k$ has bounded diameter arboricity at most $k+1$. We prove this conjecture for $k \in \{2, 3\}$ by proving the stronger assertion that the union of a forest and a star forest can be partitioned into two forests of diameter at most 18. This is joint work with Martin Merker.

## On graphs that do not contain a theta nor a wheel

Marko Radovanović

**Abstract**

A theta is a graph made of three paths, disjoint except at their ends. A wheel is a graph made of a hole plus a vertex that has at least three neighbors in this hole. In this talk we present a structure theorem for the class of graphs that do not contain a theta nor a wheel as induced subgraphs, from which we derive a polynomial time recognition algorithm for this class. As another consequence, we present polynomial time algorithms for coloring and finding a maximal stable set for graphs in this class.

This is joint work with N. Trotignon and K. Vušković.

## News on the list $k$-colorability problem in $H$-free graphs

Oliver Schaudt

**Abstract**

In the list $k$-colorability problem, each vertex of the input graph is equipped with a list that is a subset of $\{1, ..., k\}$, and the task is to decide whether there is a proper coloring that assigns to each vertex a color from its list. This generalizes the classical $k$-colorability problem and the precoloring extension problem.

I will explain our recent results on list 3-coloring in $H$-free graphs, and the methods used to prove these results. A focus will be given to the problem of characterizing the minimal obstructions against list 3-colorability.

Based on joint work with Flavia Bonomo, Maria Chudnovsky, Jan Goedgebeur, Peter Maceli, Maya Stein, and Mingxian Zhong.

## Induced subgraphs of graphs with large chromatic number

Alex Scott

**Abstract**

What can we say about the induced subgraphs of a graph $G$ with very large chromatic number? If $G$ has no large cliques, then what else can we guarantee? We will survey recent work on this topic, and present some new results. Joint work with Maria Chudnovsky and Paul Seymour.

## Banana trees and $\chi$-boundedness

Paul Seymour

**Abstract**

A "banana" is the union of a set of paths all with the same ends but otherwise disjoint; and a "banana tree" is obtained from a tree by replacing every edge by a banana. We recently proved (joint with Alex Scott and Maria Chudnovsky) that for any banana tree $H$, the class of graphs such that no induced subgraph is a subdivision of $H$ is "chi-bounded", that is, for every graph in the class, its chromatic number is at most some function of its clique number (where the function depends on the class but not on the graph).

This contains Scott's 1997 theorem that for any tree $T$, the class of graphs such that no induced subgraph is a subdivision of $T$ is chi-bounded; and our recent theorem (a 1985 conjecture of Gyarfas) that the class of graphs with no induced cycle of length more than $k$ is chi-bounded, for any fixed $k$.

We sketch the proof and survey some other results of the same type. Joint work with Alex Scott and Maria Chudnovsky.

## Approximately coloring graphs without long induced paths

Sophie Spirkl

**Abstract**

It is known that for any graph $H$ which is not an induced subgraph of a path, the $k$-coloring problem for $H$-free graphs is NP-hard for any fixed $k \geq 3$. The complexity of $k$-coloring graphs with no induced $t$-vertex path is known in all cases except when $k = 4$ and $t = 6$, or when $k = 3$ and $t \geq 8$.

We consider the following approximate version for $k = 3$: Given a graph $G$ wit no induced $t$-vertex path, find an $f(t)$-coloring of $G$ or decide that $G$ is not 3-colorable. We present an algorithm that solves this problem with $f(t) \approx t$ in general, and with $f(t) \approx t/2$ if $G$ is triangle-free.

Joint work with Maria Chudnovsky, Oliver Schaudt, Maya Stein and Mingxian Zhong.

# Tree embeddings with minimal degree conditions

Maya Stein

**Abstract**

The theme of this talk is to determine degree conditions that ensure that a given host graph obeying these conditions contains every tree of a fixed size. For instance, it is well known and easy to see that each graph $G$ of minimum degree at least $k$ contains every tree $T$ with $k$ edges. The famous Erdős–Sós conjecture suggests that we can replace the minimum degree condition with the condition that the graph has average degree greater than $k - 1$, and still find every tree $T$ as above. The Loebl–Komlós–Sós conjecture replaces the minimum degree with the median degree, with the same output. We suggest another variant, motivated by the observation that our tree $T$ has at most one vertex of really large degree (close to $k$), and the other degrees are moderate. We conjecture that any graph of minimum degree at least $\lfloor 2k/3 \rfloor$ and maximum degree at least $k$ contains every tree with $k$ edges. We prove two results in this direction, one replacing the bound on the minimum degree with a bound closer to $k$, and the other replacing the bound on the maximum degree with a bound that is exponential in $k$. This is joint work with F. Havet, B. Reed and D. Wood.

# Sparse VC-dimension

Stéphan Thomassé

**Abstract**

VC-dimension is a powerful tool which can be applied as soon as the input structure is not too complex, i.e. does not contain arbitrarily large "shattered sets". It turns out that having *no* large shattered set can be replaced by having *few* large shattered sets in the classical applications of VC-dimension (like epsilon-approximations). This observation can be applied to problems in which VC-dimension is potentially unbounded but shattered sets are somewhat controlled. For instance we can show that "If a tournament $T$ is such that every out-neighborhood has a vertex-partition into 2 transitive tournaments, then $T$ itself has a partition into $k$ transitive tournaments, for some fixed constant $k$". This question is still open when 2 is replaced by 3 or any fixed value.

# Algorithms for (cap, even-hole)-free graphs

Kristina Vušković

**Abstract**

A *hole* is a chordless cycle of length at least four, and it is *even* or *odd* depending on the parity of its length. A *cap* is a graph that consists of a hole $H$ and a node $x$ that has exactly two neighbors in $H$, that are furthermore adjacent. For a family of graphs $\mathcal{F}$, we say that a graph $G$ is *$\mathcal{F}$-free* if for every $F \in \mathcal{F}$, $G$ does not contain $F$ as an induced subgraph.

The study of cap-free graphs originated in the context of perfect graphs. A graph $G$ is *Meyniel* if every odd length cycle of $G$, that is not a triangle, has at least two chords. Note that Meyniel graphs are precisely (cap, odd-hole)-free graphs. The class was proved to be perfect in 1976 by Meyniel, and Markosyan and Karapetyan. In 1984 Burlet and Fonlupt obtained the first polynomial-time recognition algorithm for Meyniel graphs, based on their decomposition by amalgams (that they introduced). Subsequently, in 1999, Roussel and Rusu obtained a faster algorithm for recognizing Meyniel graphs (of complexity $\mathcal{O}(m^2)$), that is not decomposition based. In 1990 Hertz gave an $\mathcal{O}(nm)$ algorithm for coloring and obtaining a largest clique of a Meyniel graphs. This algorithm is based on contractions of even pairs. In 2001 Roussel and Rusu gave an $\mathcal{O}(n^2)$ algorithm that colors a Meyniel graph without using even pairs. This algorithm "simulates" even pair contractions and it is based on lexicographic breadth-first search and greedy sequential coloring.

In 1999, Conforti, Cornuéjols, Kapoor and Vušković generalized Burlet and Fonlupt's decomposition theorem for Meyniel graphs to the decomposition by amalgams of all cap-free graphs. Since triangle-free graphs are cap-free, it follows that the problem of coloring and finding the size of a largest independent set are both NP-hard for cap-free graphs. Recently, Conforti, Gerards and Pashkovich (2015) showed how to obtain a polynomial-time algorithm for solving the maximum weighted independent set problem on any class of graphs that is decomposable by amalgams into basic graphs for which one can solve the maximum weighted independent set problem in polynomial time. In particular, they show that maximum weight independent set problem can be solved in polynomial time for (cap, even-hole)-free graphs.

In this talk we present recent results on (cap, even-hole)-free graphs obtained in joint work with Kathie Cameron, Shenwei Huang and Murilo V. G. da Silva. We use known decomposition theorems to obtain an explicit construction of (cap, even-hole)-free graphs, from which we derive that every such graph $G$ has a vertex of degree at most $\frac{3}{2}\omega(G) - 1$, and hence $\chi(G) \leq \frac{3}{2}\omega(G)$. We further use structure to show that every (triangle, even-hole)-free graph has tree width at most 5. It follows that every (cap, even-hole)-free graph that has no universal vertex nor a clique cutest, has clique width at most 48. Consequently a number of problems can be solved in polynomial time for (cap, even-hole)-free graphs, and in particular, the class can be coloured in polynomial time. We also give a combinatorial algorithm for maximum weight stable set problem for (cap, even-hole)-free graphs that runs in $\mathcal{O}(mn^2)$.

# Three-coloring and list three-coloring of graphs without induced paths on seven vertices

Mingxian Zhong

**Abstract**

We present a polynomial time algorithm that determines if an input graph containing no induced seven-vertex path is 3-colorable. This affirmatively answers a question posed by Randerath, Schiermeyer and Tewes in 2002. Our algorithm also solves the list-coloring version of the 3-coloring problem, where every vertex is assigned a list of colors that is a subset of $\{1, 2, 3\}$, and gives an explicit coloring if one exists.