

Monday, December 7

9:30 - 10:30 Michele Conforti

10:30 - 11:00 Coffee Break

11:00 - 11:30 Bernard Ries, "Some properties of edge intersection graphs of single bend paths on a grid"

11:30 - 12:00 Pierre Charbit, "Linear Time Split-Decomposition of undirected graphs"

12:00 - 3:30 Lunch and discussion

3:30 - 4:00 Nicholas Trotingnon, "The  $k$ -in-a-tree problem for graphs of girth at least  $k$ "

4:00 - 4:30 Maria Chudnovsky, "A splitter theorem for induced subgraphs"

4:30 - 5:00 Coffee Break

5:00 - Problem session

Tuesday, December 8

9:30 - 10:30 Fedor Fomin, "Bidimensionality and kernelization"

10:30 - 11:00 Coffee Break

11:00 - 11:30 Dimitrios Thilikos, "Bidimensionality and contraction obstructions for treewidth"

11:30 - 12:00 Daniel Marx, "Important separators and spiders"

12:00 - 3:30 Lunch and discussion

3:30 - 4:00 Saket Saurabh, "Chromatic coding"

4:00 - 4:30 Archontia Giannopoulou, "Forbidden graphs for tree-depth"

4:30 - 5:00 Coffee break

5:00 - 5:30 Bert Gerards, "Binary matroid minors"

5:30 - 6:00 Zhentao Li, "Recognizing a totally odd  $K_4$ -subdivision and related problems"

Wednesday, December 9

9:30 - 10:30 Paul Seymour, "Tournament cutwidth"

10:30 - 11:00 Coffee Break

11:00 - 11:30 Robin Thomas, "Color-critical graphs have logarithmic circumference"

11:30 - 12:00 Reinhard Diestel, "Rayless graphs have unfriendly partitions"

12:00 - Lunch and excursion to Ravenna

Thursday, December 10

9:30 - 10:30 Bojan Mohar, "Are there some easy instances for the crossing number minimization?"

10:30 - 11:00 Coffee Break

11:00 - 11:30 Jan Krachovil, "Complexity of drawing partially embedded graphs"

11:30 - 12:00 Carl Yegerer, "A short proof on color-critical graphs on surfaces"

12:00 - 3:30 Lunch and discussion

3:30 - 4:00 Yori Zwols, "Fractionally and integrally co-strongly perfect claw-free graphs"

4:00 - 4:30 Sang il Oum, "Perfect matchings in claw-free cubic graphs"

4:30 - 5:00 Coffee Break

5:00 - 5:30 Frantisek Kardos, "Superlinear bound for the number of perfect matchings in cubic graphs"

5:30 - 6:00 Joonkyung Lee, "Rank-width of Random Graphs"

Friday, December 11

9:30 - 10:00 Dan Kral, "Removal Lemma for systems of linear equations"

10:00 - 10:30 Eli Berger, "Cake slicing, path counting and wait-free synchronizing"

10:30 - 11:00 Coffee Break

11:00 - 11:30 Ken-ichi Kawarabayashi, "An overview of the unique linkage theorem and the graph minors algorithm"

11:30 - 12:00 Paul Wollan, "A shorter proof of the unique linkage theorem"

12:00 - Lunch and departure

## Cake slicing, path counting and wait-free synchronizing

Eli Berger, Haifa University

Suppose  $n$  people  $p_1, \dots, p_n$  want to share a cake between them, namely, they want to agree on numbers  $x_1, \dots, x_n$  where  $x_i$  is the part of the cake that  $p_i$  gets. (So each  $x_i$  should be between 0 and 1 and the sum should be 1.) We assume they need to overcome two problems. The first is that their communication is not synchronized. One person can transmit a message to all the other, but there is no way to be sure who got it yet and when. The second problem is that some of the  $n$  people might not participate at all, in which case we demand that their share is zero. In this talk I will show a way to do this. More precisely, I will show an infinite algorithm in which the ideas of the people about the how the cake should be shared *converge into an agreement*. The main tool in this algorithm is the construction of a graph and counting paths in it.

I will then show how this relates to more “mainstream” computer science, where algorithms should terminate after some finite period of time. Problem of this sort are called “wait-free synchronization problems”. I will use the cake slicing algorithm in order to obtain an alternative (possibly simpler) proof of the famous result of Herlihy and Shavit saying that for problems of this sort there exists a solution if and only if some topological property holds, which resembles the Knaster - Kuratowski - Mazurkiewicz (KKM) structure.

If time allows, I will discuss another corollary, saying that in a wait-free synchronization problems one can make a “write together - read together” assumptions about the scheduling, namely that each time there is a set of people transmitting their messages together, and then each person in this set receives all messages “in the air”.

I also suggest a way of putting the Herlihy-Shavit theorem and the KKM theorem on the two ends of a continuum of conjectures, proved for a special case.

## Linear Time Split-Decomposition of undirected graphs

Pierre Charbit, LIAFA University of Paris 7

A split (or 1-join) is a partition of the vertices of a graph such that the cut is a complete bipartite between the two attachments sets. The theory of splits comes originally from the work of W.H. Cunningham (1982) and has been since that intensively studied. Some of its generaliazation, like the 2-join appear for example in the perfect graph decomposition. Here we will discuss a new and relativey simple algorithm to find the tree decomposition of splits in  $O(m)$  time. This algorithm is based on a structural result about splits and on a result of algorithmical set theory.

This is joint work with F. de Montgolfier and M. Raffinot

## A splitter theorem for induced subgraphs

Maria Chudnovsky, Columbia University

A homogeneous set in a graph  $G$  is a subset  $X$  of  $V(G)$ , such that no vertex of  $V(G) \setminus X$  has

both a neighbor and a non-neighbor in  $X$ . Let us say that a graph is prime if it has no homogeneous set  $X$  with  $1 < |X| < |V(G)|$ .

Seymour's well-known splitter theorem states that if  $G$  and  $H$  are 3-connected graphs,  $G$  is not a wheel,  $H$  is not the complete graph on four vertices, and  $H$  is a minor of  $G$ , then  $G$  can be built from  $H$  by undeleting or uncontracting one edge at a time, and so that all the graphs constructed along the way are 3-connected. We prove a similar result for the induced subgraph containment relation, replacing 3-connected with prime, undeleting and uncontracting with adding a vertex, and wheels with a certain family of bipartite graphs that we call  $B$ . We prove that if  $G$  and  $H$  are prime graphs,  $G$  is not in  $B$ , and  $H$  is an induced subgraph of  $G$ , then  $G$  can be built from  $H$ , adding one vertex at a time, and so that all the graphs constructed along the way are prime.

We then use this result to prove that every  $n$ -vertex prime claw-free graph has at most  $n + 1$  simplicial cliques (a clique  $C$  of  $G$  is simplicial if for every vertex  $c$  of  $C$ , the set of neighbors of  $c$  outside of  $C$  is a clique). This allows us to test in polynomial time if a claw-free graph has a simplicial clique, answering a question of Prasad Tetali.

This is joint work with Paul Seymour.

## Rayless graphs have unfriendly partitions

Reinhard Diestel, University of Hamburg

A bipartition of the vertex set of a graph is *unfriendly* if every vertex has at least as many neighbours in the other class as in its own. The *unfriendly partition conjecture* said, originally, that every graph has such a partition. This is easy for finite graphs, false for very large uncountable graphs, and one of the best-known infinite graph problems for countable graphs. We proved the conjecture for rayless graphs of any cardinality; I shall indicate a proof of the countable case.

This is joint work with Bruhn, Georgakopoulos and Sprüssel

## Bidimensionality and kernelization

Fedor Fomin, University of Bergen

Bidimensionality theory was introduced by Demaine et al. as a tool to obtain sub-exponential time parameterized algorithms for bidimensional problems on H-minor free graphs. Demaine and Hajiaghayi extended the theory to obtain polynomial time approximation schemes (PTASs) for bidimensional problems. In this talk, I discuss a third meta-algorithmic application of bidimensionality theory—kernelization. The talk is based on the joint work with Daniel Lokshtanov, Saket Saurabh and Dimitrios Thilikos.

## Binary matroid minors

Bert Gerards, Centrum Wiskunde & Informatica

Recently Jim Geelen, Geoff Whittle and I proved that binary matroids are well-quasi-ordered

by minors. This extends Robertson and Seymour's theorem that graphs are well-quasi-ordered by minors. Our result relies on a structure theorem for minor-closed classes of binary matroids that we completed in 2008. During this talk I will outline this structure.

This is joint work with Jim Geelen and Geoff Whittle.

## Forbidden graphs for tree-depth

Archontia C. Giannopoulou, National and Kapodistrian University of Athens

For every  $k \geq 0$ , we define  $\mathcal{G}_k$  as the class of graphs with tree-depth at most  $k$ , i.e. the class containing every graph  $G$  admitting a valid colouring  $\rho : V(G) \rightarrow \{1, \dots, k\}$  such that every  $(x, y)$ -path between two vertices where  $\rho(x) = \rho(y)$  contains a vertex  $z$  where  $\rho(z) > \rho(x)$ . In this paper we study the set of graphs not belonging in  $\mathcal{G}_k$  that are minimal with respect to the minor/subgraph/induced subgraph relation (obstructions of  $\mathcal{G}_k$ ). We determine these sets for  $k \leq 3$  for each relation and prove a structural lemma for creating obstructions from simpler ones. As a consequence, we obtain a precise characterization of all acyclic obstructions of  $\mathcal{G}_k$  and we prove that there are exactly  $\frac{1}{2}2^{2^{k-1}-k}(1 + 2^{2^{k-1}-k})$ . Finally, we prove that each obstruction of  $\mathcal{G}_k$  has at most  $2^{2^{k-1}}$  vertices.

This is joint work with Zdeňek Dvořák and Dimitrios M. Thilikos.

## Superlinear bound for the number of perfect matchings in cubic graphs

Frantisek Kardos, Charles University

Lovasz and Plummer conjectured in 1970s that cubic bridgeless graphs have exponentially many perfect matchings. This conjecture has been verified, in particular, for bipartite graphs by Voorhoeve, and for planar graphs by Chudnovsky and Seymour. In the general case, only linear bounds were known so far. In this talk, we present the first superlinear bound by showing that for any  $a > 0$  there exists a constant  $b > 0$  such that every  $n$ -vertex cubic bridgeless graph has at least  $an - b$  perfect matchings.

This is joint work with Louis Esperet and Daniel Kral.

## Complexity of drawing partially embedded graphs

Jan Krachovil, Charles University

We will survey several recent results about testing planarity of partially embedded graphs. The talk is based on a joint work with P. Angelini, G. di Battista, F. Frati, V. Jelinek, M. Patrignani, and I. Rutter

## Removal Lemma for systems of linear equations

Dan Kral, Charles University

We study algebraic analogues of the graph Removal Lemma. Vaguely speaking, the lemma says that if a given graph does not contain too many subgraphs of a given kind, then all the subgraphs of this kind can be destroyed by removing few edges. In 2005, Green conjectured the following analogue of it for systems of equations over integers:

For every  $k \times m$  integral matrix  $A$  with rank  $k$  and every  $\epsilon > 0$ , there exists  $\delta > 0$  such that the following holds for every  $N$  and every subset  $S$  of  $\{1, \dots, N\}$ : if the number of solutions of  $Ax = 0$  with  $x \in S^m$  is at most  $\delta N^{m-k}$ , then it is possible to destroy all solutions  $x \in S^m$  of  $Ax = 0$  by removing at most  $\epsilon N$  elements from the set  $S$ .

We prove this conjecture by establishing its variant for not necessarily homogenous systems of equations over finite fields. The core of our proof is a reduction of the statement to the colored version of hypergraph Removal Lemma for  $(k + 1)$ -uniform hypergraphs. Independently of us, Shapira obtained the same result using a reduction to the colored version of hypergraph Removal Lemma for  $O(m^2)$ -uniform hypergraphs.

This is joint work with Oriol Serra and Lluís Vena.

## Removal Lemma for systems of linear equations

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## Rank-width of Random Graphs

Joonkyung Lee, KAIST

Rank-width of a graph  $G$ , denoted by  $\mathbf{rw}(G)$  is a graph width parameter introduced by Oum and Seymour(2006). We investigate the asymptotic behavior of rank-width of a random graph  $G(n, p)$ , which is a graph on  $n$  vertices such that two vertices are adjacent with the probability

$p$  independently at random. This model of random graphs was introduced by Erdős and Rényi (1960). We show that asymptotically almost surely (i) if  $p \in (0, 1)$  is a constant then  $\mathbf{rw}(G(n, p)) = \lceil n/3 \rceil - O(1)$ , (ii) if  $1/n \ll p \leq 1/2$ , then  $\mathbf{rw}(G(n, p)) = \lceil n/3 \rceil - o(n)$ , (iii) if  $p = c/n$  and  $c > 1$  then  $\mathbf{rw}(G(n, p)) \geq rn$  for some  $r = r(c)$ , and (iv) if  $p = c/n$  and  $c < 1$  then  $\mathbf{rw}(G(n, p)) \leq 2$ . Thus, it is shown that  $G(n, p)$  has the sharp threshold  $p(n) = 1/n$  with respect to having linear rank-width. This is joint work with Choongbum Lee and Sang-il Oum.

## Important separators and spiders

Daniel Marx, Tel Aviv University

The notion of “important separators” and bounding the number of such separators turned out to be a very useful technique in the design of fixed-parameter tractable algorithms for multi(way) cut problems. I will overview these results and in addition, I will talk about how important separators can be used to show that “spiders with  $d$  legs” (that is,  $d$  internally disjoint paths connecting a vertex  $a \in A$  with  $d$  distinct vertices of  $B$ ) have the Erdős-Pósa property.

## Are there some easy instances for the crossing number minimization?

Bojan Mohar, University of Ljubljana and Simon Fraser University

The answer to the question in the title is clearly positive – the crossing number of every planar graph is zero, and this can be checked in linear time. Kawarabayashi and Reed proved that the same can be said for classes of graphs whose crossing number is bounded by a constant. We shall try to argue that apart from such instances, the crossing number minimization should be hard.

## Perfect matchings in claw-free cubic graphs

Sang il Oum, KAIST

Lovasz and Plummer conjectured that there exists a fixed positive constant  $c$  such that every cubic  $n$ -vertex graph with no cutedge has at least  $2^{cn}$  perfect matchings. Their conjecture has been verified for bipartite graphs by Voorhoeve and planar graphs by Chudnovsky and Seymour. We prove that every claw-free cubic  $n$ -vertex graph with no cutedge has more than  $2^{n/12}$  perfect matchings, thus verifying the conjecture for claw-free graphs.

## Some properties of edge intersection graphs of single bend paths on a grid

Bernard Ries, University of Warwick

We consider graphs  $G$  whose vertices can be represented as single bend paths (i.e., paths with at most one turn) on a rectangular grid such that two vertices are adjacent in  $G$  if and only if the corresponding paths share at least one edge in the grid. These graphs, called  $B_1$ -EPG graphs, have first been introduced by Golumbic, Lipstheyn and Stern. Here we show that the neighborhood of every vertex in a  $B_1$ -EPG graph induces a weakly chordal graph. From this we conclude that  $B_1$ -EPG graphs satisfy the Erdős-Hajnal property, i.e., that they contain either a large clique or a large stable set. Finally we give a characterization of  $B_1$ -EPG graphs among some subclasses of chordal graphs, namely chordal bull-free graphs, chordal claw-free graphs, chordal diamond-free graphs, and special cases of split graphs. This is joint work with Andrei Asinowski.

## Chromatic coding

Saket Saurabh, Institute of Mathematical Sciences

Alon, Yuster and Zwick developed the method of color-coding to give a  $2^{O(k)n^{O(t)}}$  time algorithm for subgraph isomorphism problem when the subgraph we are looking for has treewidth  $t$  and size  $k$ . We develop a variation of this method, called Chromatic Coding, where one is interested in just properly coloring the subgraph one is seeking for, as opposed to color coding, where every vertex gets a distinct color. Using this method we develop the first subexponential time algorithms for feedback arc set in tournaments, dense instances of betweenness and minimum quartet inconsistency. In this talk we will introduce the method of Chromatic Coding through the example of feedback arc set in tournaments.

This is a joint work with Noga Alon and Daniel Lokshtanov.

## Tournament cutwidth

Paul Seymour, Princeton

Let us say a digraph has cutwidth at most  $c$  if there is an ordering  $\{v_1, \dots, v_n\}$  of its vertex set such that for each  $i$  there are at most  $c$  edges from  $\{v_1, \dots, v_i\}$  to  $\{v_{i+1}, \dots, v_n\}$ . For tournaments, this concept is very useful; for instance, a tournament has bounded cutwidth if and only if no vertex is in many edge-disjoint directed cycles.

The use of cutwidth leads to a simple polynomial-time algorithm for the  $k$ -edge-disjoint directed paths problem for tournaments for fixed  $k$  (we are given  $k$  pairs of vertices of a tournament, and want to decide whether there are  $k$  edge-disjoint directed paths joining the pairs). This extends a result of Bang-Jensen, who found a polynomial-time algorithm for the case  $k = 2$ .

We also use cutwidth to show that in any infinite tournaments, one of them can be immersed in another ( $H$  is immersed in  $G$  if its vertices are mapped to distinct vertices of  $G$ , and its edges are mapped to edge-disjoint directed paths of  $G$  joining the appropriate pairs of vertices). This is an analogue for tournaments of the theorem of Neil Robertson and the speaker that graphs are well-quasi-ordered by minor containment.

This grew out of work on a conjecture of S.B. Rao, that degree-sequences of graphs are well-quasi-ordered by a certain containment relation, and if time permits we will explain the connection.

This is joint with Maria Chudnovsky, Columbia, and Sasha Fradkin, Princeton.



## Bidimensionality and contraction obstructions for treewidth

Dimitrios Thilikos, National and Kapodistrian University of Athens

We provide two parameterized graphs  $\Gamma_k, \Pi_k$  with the following property: For every positive integer  $k$ , there is a constant  $c_k$  such that every graph  $G$  with treewidth at least  $c_k$ , contains one of  $K_k, \Gamma_k, \Pi_k$  as a contraction, where  $K_k$  is a complete graph on  $k$  vertices. As none of these graphs (for  $k \geq 6$ ) is a contraction of the other, these three parameterized graphs can be seen as “obstruction patterns” for the treewidth with respect to the contraction quasi-ordering. As a consequence of our combinatorial results we unify and significantly simplify contraction bidimensionality theory - the meta algorithmic framework to design efficient parameterized and approximation algorithms for contraction closed parameters.

Joint work with Fedor V. Fomin and Petr. Golovach.

## Color-critical graphs have logarithmic circumference

Robin Thomas, Georgia Institute of Technology

A graph is  $k$ -critical if every proper subgraph is  $(k - 1)$ -colorable, but the graph itself is not. We prove that every  $k$ -critical graph on  $n$  vertices has a cycle of length at least  $\log n / (100 \log k)$ . Examples show the bound cannot be improved to exceed  $2(k - 1) \log n / \log(k - 2)$ . This is joint work with A. Shapira.

## The $k$ -in-a-tree problem for graphs of girth at least $k$

Nicolas Trotignon, CNRS, LIAFA, University of Paris 7

For all integers  $k \geq 3$ , we give an  $O(n^4)$  time algorithm for the problem whose instance is a graph  $G$  of girth at least  $k$  together with  $k$  vertices and whose question is “Does  $G$  contains an induced subgraph containing the  $k$  vertices and isomorphic to a tree?”.

This directly follows for  $k = 3$  from the three-in-a-tree algorithm of Chudnovsky and Seymour. Here we solve the problem for  $k \geq 4$ . Our algorithm relies on a structural description of graphs of girth at least  $k$  that do not contain an induced tree covering  $k$  given vertices ( $k \geq 4$ ).

Joint work with N. Dehry, C. Picouleau and Liu Wei.

## A shorter proof of the unique linkage theorem

Paul Wollan, University of Rome La Sapienza

Let  $G$  be a graph and  $P_1, \dots, P_k$  disjoint paths in  $G$  such that  $\bigcup_1^k V(P_i) = V(G)$ . The unique linkage theorem of Robertson and Seymour states that either there exist disjoint paths  $P'_1, \dots, P'_k$

such that  $P_i$  and  $P'_i$  have the same endpoints for  $1 \leq i \leq k$  and  $\bigcup_1^k V(P'_i) \subsetneq V(G)$ , or, alternatively, the tree width of  $G$  is bounded by a function of  $k$ .

The unique linkage theorem is a major tool in the proof of the polynomial running time of Robertson and Seymour's algorithm for the  $k$  disjoint paths problem. The original proof of Robertson and Seymour of the unique linkage theorem is quite difficult and uses the structure theorem for graphs excluding a clique minor. I will continue where Ken-ichi Kawarabayashi's talk left off and discuss a new and simpler proof that avoids many of these difficulties; specifically, the proof does not rely on the excluded minor structure theorem of Robertson and Seymour.

This is joint work with Ken-ichi Kawarabayashi.

## A short proof on color-critical graphs on surfaces

Carl Yerger, Georgia Institute of Technology

In this talk, I will describe a short proof of the well-known theorem by Thomassen which says that there are only finitely many six-critical graphs on a fixed surface with Euler genus  $g$ . In addition, I will describe some structural lemmas that were useful to the proof and describe a list-coloring extension that is helpful to ongoing work that there are finitely many six-list-critical graphs on a fixed surface. This is joint work with Ken-ichi Kawarabayashi.

## Fractionally and integrally co-strongly perfect claw-free graphs

Yori Zwols, Columbia University

Strongly perfect graphs have been studied by several authors (e.g. Berge, Duchet, Ravindra, Wang). This talk deals with a fractional relaxation of strong perfection. Motivated by a wireless networking problem, we consider claw-free graphs that are fractionally strongly perfect in the complement. We obtain a forbidden induced subgraph characterization and display graph-theoretic properties of such graphs. It turns out that the forbidden induced subgraphs that characterize claw-free fractionally co-strongly perfect graphs are precisely the cycle of length 6, all cycles of length at least 8, four particular graphs, and a collection of graphs that are constructed by taking two graphs, each a copy of one of three particular graphs, and joining them by a path of arbitrary length in a certain way. Wang gave a characterization of strongly perfect claw-free graph. We obtain as a corollary claw-free graphs whose complements are strongly perfect. This is joint work with Maria Chudnovsky and Bernard Ries.